Complete FIVE problems

Problem 1

Explain what Jeffrey conditionalization is.

Problem 2

This is the AXIOM OF REGULARITY in standard (ZF) set theory (the quantifiers range over all sets):

$$\forall x \neq \emptyset \rightarrow \exists y \in x(y \cap x = \emptyset)$$

Show that it follows from the axiom of regularity that there is no set A such that $A \in A$.

Problem 3

Recall that in the notes we 'defined' ordered pairs in set-theoretic terms as follows: $\langle x, y\rangle = \{x, \{x, y\}\}$. Consider an alternative definition $\langle x, y\rangle = \{x, \{y\}\}$. What is the problem with this definition?

Problem 4

There are two coins, one is a fair coin, the other is two-headed. I am randomly given one of these coins and I flip the coin and it lands heads. What is the probability the coin I am given is fair?

Turn over

Problem 5

Recall the classification of models of modal logic, defined by the constraints on the accessibility relations:

- K: R is any relation
- T: R is reflexive
- B: R is reflexive and symmetric
- S4: R is reflexive and transitive
- S5: R is reflexive, transitive and symmetric

For this formula of modal logic: $\Diamond P \rightarrow \Box\Diamond P$ say on which of these 5 classes it is valid. Given counter-models and validity proofs to justify your claims (but no more than necessary!).

Problem 6

Recall the strong Kleene trivalent truth tables. Suppose we describe the logical consequence relation as follows: for any sentences α and β, $\alpha \models_S \beta$ iff for every valuation v that makes α true (i.e. not false or truth-valueless) v also makes β true. Give an example of two sentences ϕ and ψ such that $\phi \models \psi$ (in the classical sense) but $\phi \not\models_S \psi$.

Problem 7

Consider a model of epistemic logic $\langle W, R_K, R_B, I\rangle$ for a knowledge relation K and a belief relation B. wR_Kw' means it is not known (by the agent) at w that w' does not obtain, while wR_Bw' means it is not believed (by the agent) at w that w' does not obtain. As usual $w \models K\phi$ iff for all w' if wR_Kw' then $w' \models \phi$; $w \models B\phi$ iff for all w' if wR_Bw' then $w' \models \phi$.
State a joint condition on R_K and R_B that will make the following axiom scheme valid:
(Knowledge entails belief) $K\phi \rightarrow B\phi$

 END