Presupposition Projection and Logical Equivalence*

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1 Introduction

A typical use of any of the three sentences below would take for granted that Mary was a communist.

(1) a. John knew Mary was a communist.
    b. John didn't know Mary was a communist.
    c. If John knew that Mary was a communist, he'd have fired her.

These sorts of observations stretch back, in their contemporary form, to the discussion of definite descriptions in Strawson (1956). On the Strawsonian style of analysis—which was not explicit but very suggestive—these judgments are explained by the fact that "x knows P" is an expression that requires for its meaningfulness the truth of P. This is one possible explanation of this phenomenon but not the only one, and I will take the subject of *presupposition* to be the phenomenon illustrated by (1), without tying the term to any particular style of explanation. When I talk of *presuppositions*, I refer to the propositions that utterances of certain sentences take for granted, in the way that the utterances of the sentences in (1) take for granted that Mary was a communist. On this way of talking about presuppositions, the question is not whether presuppositions exist but how to explain them.

There are several research questions that arise in the study of presupposition. One prominent question, the *triggering* question, is: why do certain terms such as definite descriptions, factive verbs, and expressions such as *stop*, *still*, *too*, and *also* give rise to presuppositions? Another question involves patterns such as that exemplified by

*Forthcoming in *Philosophical Perspectives*. I am grateful to Be Birchall, Jessica Boyd, Emmanuel Chemla, Ben George, Nathan Klinedinst, and Philippe Schlenker for comments and discussion.

(1). This question of *presupposition projection* is: when and how do complex sentences inherit the presuppositions of their parts? A satisfactory account of presupposition must answer both of these questions (and more).

Ideally, these questions should be answered in tandem with one grand theory of presupposition (or, even better, one grand theory of something else which explains away presupposition). Thirty years of serious work on presupposition suggests, though, that we ought to lower our sights a bit. We might try to make progress on one question while leaving others unanswered for the time being. This paper is an effort in this vein. I will address the problem of presupposition projection while remaining silent about the triggering problem and I will have almost nothing conclusive to say about what presuppositions ultimately are. I will also restrict myself to discussing propositional fragments and so exclude any discussion of quantified presuppositions.

Even this small corner of the world of presuppositions has its interest. In this paper, I will review some of the basic data about presupposition projection and discuss some of the theoretical descriptions of the this data. One point at issue here is whether presupposition projection is *symmetrical* with respect to commutative operators; for instance, whether the presuppositions of a sentence of the form $A \wedge B$ are the same as those of a sentence of the form $B \wedge A$. I will argue a) that theories that posit asymmetries in presupposition projection across commutative connectives make several incorrect predictions, and b) that even if some asymmetries exist they might be treated as processing effects rather than part of the basic pattern of presupposition projection. I will then review some recent symmetric theories of presupposition. I will suggest that the current theories, as well as the empirical data, make plausible an overarching principle about presupposition projection: *the presuppositions of complex sentences are invariant over a wide-class of logical equivalences that do not involve tautologies or contradictions.*

2 Basic Data

2.1 Trigger and Presupposition

A *presupposition trigger* is an expression whose use generally give rise to an inference that something is taken for granted. The two examples I use throughout this paper are the factive verb *know* and the verb *stop*.[1] We will say that *x knows that S* has

[1] We should not assume that all triggers behave the same. It may well be that definite descriptions, for instance, trigger different sorts of presuppositions from these two expressions.

the presupposition *S* and *x stopped F-ing* has the presupposition *x used to F*. This is reflected by the fact that uses of the two a. sentences below tend to take the b. sentences for granted:

(2) a. John knows Mary is pregnant.
 b. Mary is pregnant.

(3) a. Bill stopped smoking.
 b. Bill used to smoke.

There are contexts where (2-a) or (3-a) could be used without taking for granted (2-b) and (3-b) but in general they do take them for granted. For the rest of this paper, I will ignore such instances of *cancellation*, i.e., instances when a putative presupposition doesn't appear in some context: I am interested in the pattern of default inferences about what is being taken for granted not the instances where the default is overridden.[2]

2.2 Projection from Connectives

The real mark of presupposition, however, is not just this sense of taking something for granted, but rather that this sense is preserved when simple sentences that trigger presuppositions are embedded in complex sentences. For instance, the negations of the a. sentences above, (4) and (5) still tend to take the b. sentences above for granted.

(4) John doesn't know Mary is pregnant.

(5) Bill didn't stop smoking.

There will be some uses of (4) and (5) where (2-b) and (3-b) are not taken for granted, but, by and large, using these complex sentence leads to the same presuppositions as using the simple sentences does.

[2]The notion of context here must be broad enough to encompass instances of cancellation of a presupposition in a single sentence across all contexts because the sentence itself forces a feature of the context that cancels the presupposition. For example, *I don't know that John smokes because he doesn't smoke*. Of course, cancellation is a phenomenon that all theories of presupposition must ultimately deal with, but it is not something that obviously needs to be discussed in the context of discussing presupposition projection. Gazdar (1979) and Soames (1976, 1982) give extensive treatments of cancellation: Gazdar tries to treat all presupposition projection as instances of cancellation, but Soames gives compelling arguments that one needs to have separate rules for projection and cancellation even if there is some overlap.

In all our examples, so far, a complex sentence has simply *inherited* the presuppositions of its atomic parts. This is not always the case, however. In Karttunen's terminology presuppositions can be *filtered* out in certain contexts (Karttunen, 1974). The sentences in (6) and (7), below, are complex sentences that include (2-a) and (3-a), respectively, but do not seem to have any presuppositions:

(6) a. If John got Mary pregnant, John knows Mary is pregnant.
b. Either Mary isn't pregnant, or John knows Mary is pregnant.
c. Maybe Mary is pregnant and John knows it.

(7) a. If Bill used to smoke, he stopped smoking
b. Either Bill didn't used to smoke, or he stopped smoking.
c. Maybe Bill used to smoke and he stopped.

2.3 Example: The Presuppositions of Conditionals

If A is an expression with no presupposition and B is an expression with a presupposition $P(B)$, then what is, generally speaking, the presupposition of $A \rightarrow B$? The dominant hypothesis is as follow:

Cond The presupposition of $A \rightarrow B$ (as specified above) is $A \rightarrow P(B)$.

I will give two pieces of evidence in favor of **Cond**. First, if A entails $P(B)$ then, in general, the whole sentence has no presupposition. As an example consider (6-a) above, repeated here as (8):

(8) If John got Mary pregnant, John knows Mary is pregnant.

If A = *John got Mary pregnant* and B = *John knows Mary is pregnant*, and, thus $P(B)$ = *Mary is pregnant*, then the presupposition of (8) on **Cond** is $A \rightarrow P(B) = \top$. So, **Cond** correctly predicts that (8) has no presupposition despite containing an expression with a non-trivial presupposition.

Second, when A does not entail $P(B)$, what is taken for granted often seems to be $A \rightarrow P(B)$. Here is one example, where A = *John tested positive* and $P(B)$ = *John has the disease*:

(9) If John tested positive, then he knows he has the disease.

A use of (9) seems to presuppose the non-tautologous proposition *if John tested positive then he has the disease*, which is what **Cond** predicts.

There are many challenges to the idea that such conditional presuppositions are really observed (e.g. Geurts, 1996). But the conditional presuppositions view (which, as we shall see, can be extended to conjunctions and disjunctions) seems to me to be the best hypothesis (for a recent defense see van Rooij, 2007).

2.4 Compositional Rules

Reasoning such as that of the previous section allows the formulation of general rules for how complex sentences inherit the presuppositions of their parts. Long dominant in the literature have been such *heritage* rules developed by Karttunen (1974). These rules essentially form a compositional semantics for the presuppositions of complex sentences based on the presuppositions of their parts. Below is one standard version of these rules as they are applied to logical operators:

Karttunen Heritage Rules Suppose A and B are any atomic or complex sentences and $P()$ is a function from sentences to presuppositions. Then P has the following properties:

- $P(\neg A) = P(A)$
- $P(A \wedge B) = P(A) \wedge (A \rightarrow P(B))$
- $P(A \vee B) = (P(A) \wedge (\neg A \rightarrow P(B))$

We can expand our rules to encompass various other non-logical expressions. For instance, forming a sentence into a question, just like negating a sentence, does not seem to affect presuppositions. Many other non-logical sentential operators such as *it's unlikely* and *possibly* are generally thought to act like negation with respect to presuppositions. These expansions of the rules can be stated as follows:

- $P(\text{it's unlikely that } A) = P(A)$
- $P(\text{possibly } A) = P(A)$
- $P(A?) = P(A)$

2.5 Symmetries

A surprising feature of the rules above is that the heritage rules for the commutative operators $\wedge$ and $\vee$ are not themselves commutative. We might expect that $P(A \wedge B) = P(B \wedge A)$, since $A \wedge B$ and $B \wedge A$ are themselves logically equivalent, and, similarly, we might expect that $P(A \vee B) = P(B \vee A)$. I'll call heritage rules that satisfy this principle for conjunction and disjunction *symmetric* heritage rules.

The motivation for the asymmetric heritage rules, naturally, is empirical rather than conceptual. The usual way in which the data is described is that a presupposition trigger can have its presupposition filtered by something that comes before it in a sentence, but not by something that comes after it. Thus, a presupposition in the second conjunct may be filtered out by something in the first, but not vice-versa. So, for instance, the following a. sentence is meant to have a presupposition (that Mary is a communist) but the b. sentence is not meant to:

(10) a. Joe knows Mary is a communist and Mary is a communist.
b. Mary is a communist and Joe knows it [that Mary is a communist].

This contrast in presupposition projection is perhaps best seen by putting these sentences in the form of a question, which allows us to distinguish between a presupposition and a mere entailment—since questions do not have entailments. Compare these two questions:

(11) a. Is it true that Joe knows Mary is a communist and Mary is a communist?
b. Is is true that Mary is a communist and Joe knows it?

My sense of the data is that in both (10) and (11) the a. sentence sounds much odder than the b. sentence. This, at least, shows that something goes wrong when material that would filter a presupposition comes after rather than before the presupposition trigger.

A symmetric theory of presupposition does not have many resources to explain the asymmetry demonstrated above. Consider, for instance, this symmetric rule for presupposition projection in conjunctions.

- $P(A \wedge B) = (P(A) \wedge (A \rightarrow P(B))) \vee (P(B) \wedge (B \rightarrow P(A)))$

On this rule, both the a. sentences and the b. sentences above would have no presuppositions. In this case we would have no foothold for explaining why the a. sentences are more awkward than the b. sentences. This sort of reasoning might lead one to prefer asymmetric rules. (The data is less clear-cut with *or* and, indeed, many early theorists, such as Soames (1982), argued that the rule for *or* should be symmetric.)

Recent work has challenged the consensus in favor of asymmetric theories.[3] The cases required to motivate symmetric rules of presupposition projection are slightly more complex than the very standard cases, but I think the judgments are relatively clear. The reason we need to look at complex cases is that there may be independent pragmatic principles interfering with our judgments in many simple cases. For example, the reason sentences of the form $A \wedge P(A)$ are unacceptable may be that there is a general prohibition against saying sentences of the form $A \wedge B$ if A entails B (but not vice versa). We can call this sort of prohibition an asymmetric *anti-redundancy* principle.[4] In support of this kind of principle, Schlenker notes that when the first conjunct entails the second in a conjunction, as in (12-a), there is a pragmatic oddness, but when the weaker conjunct comes first, as in (12-b), there is no oddness.

(12) a. ?John is a practicing, accredited doctor and he has a medical degree.
b. John has a medical degree and he is a practicing, accredited doctor.

If we assume, in general, that A entails $P(A)$, then every sentence of the form $A \wedge P(A)$ violates the anti-redundancy principle. So, that gives an independent explanation of why sentences such as (10-a) and (11-a) sound odd that has nothing to do with presupposition. This suggests that we need to look at more complex examples to judge whether symmetric or asymmetric theories better capture the data.

When we control for independent pragmatic facts like the anti-redundancy principle, we find many sentences where the asymmetric rules of the previous section make false predictions. For example, consider this sentences which does not violate the anti-redundancy principle:

(13) If John doesn't know it's raining and it is raining heavily, then John will be surprised when he walks outside.

[3] See in particular Schlenker (to appear, 2008) though his analysis differs from mine.

[4] I'm not sure how general the most plausible version of the principle is. It might apply under some embeddings, such as in questions, but my sense is that it does not apply under negation or in the antecedent of conditionals.

The form of this sentence is $(A \wedge B) \rightarrow C$. Clause A (= *John doesn't know it's raining*) has a presupposition $P(A)$ = *it's raining*, and, for our purposes, we can assume that B and C have no presuppositions. So, B (= *it's raining heavily*) entails $P(A)$. On the Karttunen projection rules, the presupposition of this entire complex is $P(A)$. It seems to me that the most natural judgment is, however, that a use of (13) sentence would make no presupposition. As we shall see in the next section, a symmetric treatment of $\wedge$ will yield this judgment. Here are two more examples in this vein:

(14) Is it true that that Bill doesn't know Mary was fired, but she was fired yesterday?

(15) Either John stopped smoking or he never smoked much.

The reader can verify that (14) and (15) are both also predicted to yield presuppositions on the asymmetric Karttunen rules, despite the fact that both sentences don't seem to give rise to presuppositions.

The discussion above lends some plausibility to the notion that all the observed asymmetries in presupposition projection are due to independent pragmatic factors. There is some evidence against this, however. For example we can there may be conjunctive sentences that don't violate the anti-redundancy principle which nonetheless support asymmetric heritage rules:

(16) Is it the case that John stopped smoking but he used to smoke very heavily?

Some report that (16) most naturally presupposes that John used to smoke. I am not sure what exactly the status of the data on this point is.

However, even if presupposition projection exhibits some asymmetries that do not have an independent pragmatic explanation, we may still wish to posit symmetric heritage rules. For the asymmetries might be processing effects rather than part of the fundamental rules of projection. The existence of examples along the lines of (13) through (15), the proper treatment of which seems to require symmetric rules, makes it more appealing to have a system which both posits symmetric rules and some asymmetries on top of these, rather than a simple asymmetric system. I will discuss later, how, once we have symmetric rules, we can posit processing effects that yield asymmetric judgments.[5]

[5] This observation builds on some of Schlenker's, though his nuanced take on symmetries differs substantially from mine.

2.6 Symmetric Heritage Rules

A symmetric theory of presupposition projection satisfies the following conditions:

- $P(A \wedge B) = P(B \wedge A)$
- $P(A \vee B) = P(B \vee A)$

How do we generate plausible heritage rules that satisfy these conditions? A simple idea for conjunction is to take Karttunen's rules for $A \wedge B$ and $B \wedge A$ and disjoin them. Applying this strategy generally we get the following rules:

- $P(\neg A) = P(A)$
- $P(A \wedge B) = (P(A) \wedge (A \rightarrow P(B))) \vee (P(B) \wedge (B \rightarrow P(A)))$
- $P(A \vee B) = (P(A) \wedge (\neg A \rightarrow P(B))) \vee (P(B) \wedge (\neg B \rightarrow P(A)))$

These rules capture the data of presupposition projection fairly well. For example, these rules correctly predict that (13) through (15) don't have any presuppositions as well as handling all the previous examples.[6]

3 Generalizing and Explaining Heritage Rules

Providing symmetric heritage rules allows us to predict what the presuppositions of arbitrarily complex sentences are, as long as we know what the presuppositions of their parts are. However, we might still want to know what the underlying generalizations governing the rules are and why the rules obtain. In particular, once rules have been settled on one might still ask these two questions:

1. How can we tell for any arbitrary truth-functional connective $*$ what $P(A * B)$ is in terms of P(A) and P(B)?
2. Why do presuppositions project according to the rules?

[6] I leave discussion of conditionals out here and for much of the rest of the paper. The symmetry conditions do not apply to conditionals. Certain principles I adopt later make very specific predictions about conditionals, *if* they have a truth-functional meaning. The problem is that conditionals are probably not truth-functional in the usual sense, and even if their meaning can be approximated by truth-functions they clearly trigger implicatures of some sort, which can get in the way of making clear predictions about their projection behavior.

Question 1 is simply the question of how, in the arbitrary case of a truth-functional operator, it behaves with respect to presupposition projection. Of course, a set of heritage rules will specify how the three standard connectives (as well as negation) compositionally project presuppositions. We might want to know more than this. The standard Karttunen rules have nothing to say on this point. For example, supposing *unless* were truth-conditional, it would be nice to have a theory that yields a prediction of how it would then treat presupposition.[7] Moreover, an answer to question 1 would ideally provide a simple formula for the standard connectives that goes from their truth-table meaning (assuming they are truth-functional) to their heritage rules, so that we do not have to separately list them for each connective.[8]

Question 2 is more straightforward, if harder to answer. Karttunen's account is essentially descriptive. Ideally we should have an explanation of why the rules are as they are. It might also be hoped that such an explanation would yield an answer to question 1. An answer to question 2 might depend on answering broader questions about presuppositions generally, but it also might not.

There are two, related strands of answers to these questions: the pragmatic story developed by Stalnaker (and Soames, 1982) and the program of dynamic semantics developed by Heim and others. Each of these stories attempted to explain heritage rules akin to those of Karttunen's in section 2.4.

Soames (1989), Heim (1990), and Schlenker (to appear) review and criticize both these approaches, and I will review some of their conclusions in my discussion. First, though, I will present the *common ground* framework for describing presupposition.

4 Older Theories

4.1 Methodological Note: Common Grounds

One of the marks of linguistic presuppositions is that when a sentence presupposes another sentence an assertion of the sentence seems to take the other for granted. We thus might describe presuppositions by saying that a sentence A presupposes another sentence $P(A)$ if an assertion of A is only acceptable in a context in which the mutual assumptions of the conversation participants include $P(A)$. This framework, borrowed

[7]Schlenker (to appear, 2006) presses this point against Heim's theory.

[8]Of course, some such a formula must exist, but we would hope that it would be fairly simple.

from Stalnaker (1974), takes presupposition to be conditions on the *common ground*, the collection of mutually accepted assumptions between conversational participants.

Here is a more careful explanation of the framework: in a conversation any utterance is made against a set of possible worlds (or equivalently, a proposition) c, which is the set of worlds which are not ruled about by the mutual assumptions of the conversational participants (or the proposition true in only those worlds). When one asserts a proposition A the normal effect is, if the audience accepts the assertion, to remove all the worlds where A is false from the common ground to get a new common ground (or, in other words, to get a new common ground equivalent to $c \wedge A$). One way of working presuppositions into this framework is to assume that certain sentences such as A are such that they are only acceptably asserted in certain common grounds. In particular, we say that if A presuppose $P(A)$, then A is only acceptable if uttered in common grounds that entail $P(A)$. When it is felicitous, the effect of an assertion of A is to remove certain worlds from the common ground.

In general, we should note the following equivalence between talk in terms of a function $P()$ and talk in terms of acceptability in the common ground.

Presuppositions and Acceptability A sentence S is *acceptable* in the common ground c iff $c \models P(S)$.

We can, thus, restate the asymmetric heritage projection rules from Karttunen in terms of conditions on the common ground for the acceptability of an expression:

1. $\neg A$ is acceptable in c iff $c \models P(A)$
2. $A \wedge B$ is acceptable in c iff $c \models P(A) \wedge (A \rightarrow P(B))$
3. $A \vee B$ is acceptable in c iff $c \models P(A) \wedge (\neg A \rightarrow P(B))$
4. $A \rightarrow B$ is acceptable in c iff $c \models P(A) \wedge (A \rightarrow P(B))$

Of course, much more needs to be said to make this way of talking plausible. In particular we need a notion of *accommodation* by which the audience spontaneously adds a proposition to the common ground to make an expression acceptable in a context (Lewis, 1983; Stalnaker, 2002).

4.2 Stalnaker's Pragmatic Story

Stalnaker (1974) gave an explanation for the Karttunen projection rules in the case of conjunction and conditionals, which Soames (1982) extended to disjunction. What Stalnaker suggested is that the common ground changes as we process sentences in a way that predicts the basic Karttunen heritage rules of projection.

Consider, for instance, a conjunctive sentence where the first conjunct entails the presupposition of the second conjunct:

(17) John used to smoke, but he's stopped.

Now, if this sentence were asserted in a conversation, and you were inclined to believe the speaker, then the common ground might alter as the sentence was processed. For after, you've heard the first part of (17), *John used to smoke*, assuming you know the syntactic form of the sentence, you can add to the common ground the fact that John used to smoke (after all the speaker asserted it!). So that means that when you get to the second part, *but he's stopped*, the common ground already entails that John used to smoke. If the condition on the acceptability of *he's stopped [smoking]* is that the common ground entails that John used to smoke, then the second conjunct will always be acceptable after the first.

A similar story might also be told about conditionals. You might understand how one processes $A \to B$ in a Ramsey-like way where one first adds A to the common ground and then make sure that B follows by eliminating all the worlds where B is false once A has been added. Then one removes the supposition that A is true by putting all the $\neg A$ worlds back in the common ground. This may sound a little involved, but it isn't an unreasonable idea. If this is what actually happens, then, in a sentence of the form $A \to B$, every time B is added to the common ground A will have already been added. This ensures that a sentence such as (18) makes no presupposition:

(18) If John used to smoke, then he's stopped.

If these two stories are right, then for sentences of the form $A \wedge B$ and sentences of the form $A \to B$ we will derive the following heritage rules, which are identical to the asymmetric Karttunen rules:

- $P(A \wedge B) = P(A) \wedge (P(A) \to P(B))$

- $P(A \to B) = P(A) \wedge (P(A) \to P(B))$

Let's suppose this account works in some rough way for unembedded conditionals and conjunctions. The problem is that there isn't obviously a plausible extension of it to all complex expressions. Particular problems are disjunctions and complex expressions embedded under other operators.[9] Consider a sentence such as *If A or B, then not (if C, then D).* Can we really tell a plausible story about what the common ground will be midway into processing a sentence like this? As we will see with Heim's account when we try to extend this sort of system to a full propositional fragment, we lose some of its explanatory power.

4.3 Heim's Dynamic Semantics

Heim (1983) developed what might be considered an explicit formalization of Stalnaker's theory that presuppositions projection can be explained by the dynamics of changes in the common ground in calculating the effect of utterances.

Instead of assigning truth conditions (propositions) to sentences, she assigns rules for updating the common ground that may be undefined for some common grounds (these are called *context change potentials*, or *CCPS*). Each atomic sentence A is associated with a means of updating the common ground but the rule is only defined when the common ground entails the encoded presupposition of A, $P(A)$. When A is defined, in the propositional case, its effect on the common ground will be equivalent to intersecting the common ground with a proposition, which I'll call the *force of* A, A'.[10]

To give a concrete example: if A is *John stopped smoking*, then $P(A)$ will be *John used to smoke*. So A is only defined in a common ground c if $c \models P(A)$. When A is defined the effect on c of asserting A will be to remove all those worlds in which John smokes now. So, the force A' of A is the proposition that John doesn't smoke now. (In fact, there will be many possible forces that do the work, since, for instance, the proposition that John doesn't smoke now and John used to smoke will have the same effect on c when A is defined for c.) Following Heim we will write the result of updating

[9] Soames (1982) tells a Stalnaker style story about disjunction that derives symmetric projection rules for disjunction. He admits, however, that his and Stalnaker's account does not obviously extend to embeddings.

[10] Heim does not actually discuss forces, but without adding this constraint on to CCPS the system has much more expressive power than it should.

the common ground c with A as $c + A$. When defined, of course, $c + A$ is equivalent to $c \wedge A'$.

(We can also associate context with non-presuppositional sentences. If A is proposition then we can defined the CCP A s.t. $P(A)$ is a tautology and $A' = A$. $c + A$ then equals $c \wedge A$, which is the normal state of affairs.)

Heim's context change potentials for complex sentences are determined by the context change potentials of their parts in a compositional manner. For each complex expression of the form $A * B$, Heim defines a *update procedure* for determining its meaning based on the meaning of the CCPS, A and B. Take, as an example, $A \wedge B$. Heim defines a procedure equivalent to the changes in the common ground Stalnaker proposed:

- $A \wedge B = (c + A) + B$

What $(c + A) + B$ means is that first we apply the update procedure, A, to the common ground c, and then we take the resulting proposition and apply B to that. The result, by definition, is the effect of $A \wedge B$ on c. Heim assumes that if any part of an update procedure is undefined the entire procedure is undefined. If A and B have the presuppositions $P(A)$ and $P(B)$ then it follows that the update procedure $A \wedge B$ is defined if and only if $c \models P(A)$ and $c + A \models P(B)$. This matches the Karttunen rules from section 2.4 once we translate it back into his terminology.

Indeed, it turns out that there are update procedure for all logical connectives that yield Karttunen's rules for presupposition projection based on their definedness conditions. Here are some standard update rules, taken from Heim and others:

- $c + \neg A = c \wedge \neg(c + A)$
- $c + (A \vee B) = (c + A) \vee ((c + \neg A) + B)$
- $c + (A \rightarrow B) = (c + \neg A) \vee ((c + A) + B)$

Heim's approach, however, gives only a limited answer to question 1. For there are different and, in some sense, equally good update procedures for some of the logical operators that lead to different predictions about presupposition projection. For example. $(c + B) + A$, captures conjunctions of the form $A \wedge B$, logically speaking, as well as $(c + A) + B$ does but it makes different predictions about the behavior of presuppositions under conjunction. In particular, $(c + B) + A$ requires that $c \models P(B)$ and that $c + B \models P(A)$. On these rules (19) would be infelicitous in any common ground that did not entail that John has a son, an obviously false prediction.

(19) John has a son and Bill knows it.

Thus, Heim's dynamic semantic is not fully *explanatory* since it stipulates a particular update procedure for each connective, rather than allowing the update procedure to follow from the truth-table meaning of the connective.[11] While in the case of conjunction, Heim's procedure is perhaps more *natural* it is not true in the general case that the most natural update procedure yields the best predictions about presupposition projection.[12] Thus, to the degree that Heim's dynamic semantics predicts the inheritance properties, those predictions depend on stipulations about the exact update procedures for each of the logical operators. So Heim's account does not answer question 1 above in a satisfactory way.

5 Recent and Revived Theories

Recent work by Schlenker has inspired the invention or revival of a variety of different systems (including two of his own) for predicting and explaining presupposition projection. There are several new (or revived) theories, which, by either ignoring asymmetries or treating them as a processing effect, can capture projection rules close to symmetric versions of Karttunen's rules *and* which answer questions 1 and 2 above. I review the way these theories explain presupposition projection in the propositional case here.[13]

5.1 Strong-Kleene Logic

One classic symmetric theory of presupposition projection which turns out to make surprisingly good predictions is based on the strong-Kleene truth-tables (see Beaver,

[11]This criticism is well-known in the literature from Soames (1989) and Heim (1990, where the objection is attributed to Mats Rooth).

[12]Indeed, arguably, the most natural procedures predict almost none of the right facts. For instance, the following are very simple acceptable update procedure for disjunction and conjunction that show the parallels between them:

$$c + (A \wedge B) = (c + A) \wedge (c + B)$$

$$c + (A \vee B) = (c + A) \vee (c + B)$$

However, to make decent predictions one needs to treat conjunction as $c + A + B$. Thus, trying to preserve the natural symmetries between the dual operators, $\wedge$ and $\vee$, in the update procedure, leads to the wrong predictions.

[13]The reader might also look at Schlenker (2008) for comparisons of some of these theories.

1997; Beaver and Krahmer, 2001; George, 2008; Schlenker, 2008). These trivalent truth-tables are given below:

A	$\neg A$
T	F
F	T
*	*

A$\vee$ B	T	F	*
T	T	T	T
F	T	F	*
*	T	*	*

$A \wedge B$	T	F	*
T	T	F	*
F	F	F	F
*	*	F	*

We could use these tables to yield a system of presupposition projection in a quite straightforward way: Suppose a presuppositional sentence A is true or false in w iff $P(A)$ is true or false in w. We then assume that we can utter a proposition, A, in a common ground, c, iff A is true or false in every world in c.[14]

Take as an example:

(20) Mary knows John has a bow-tie or John doesn't have a bow-tie.

We can present this schematically as follows, letting $M =$ *Mary knows John has a bow-tie* and its presupposition $P(M) =$ *John has a bow-tie*:

(21) $M \vee \neg P(M)$

We can see that (21) will be true or false in any world since if M is undefined then $P(M)$ is false and the strong-Kleene truth table will make the whole expression false. Since (21) is true or false in every world it is acceptable in any common ground, and

[14]See Soames (1989) for discussion of why this principle is not actually as plausible as it seems.

so has no presupposition. Consider, by contrast, the sentence $\neg M$. This is neither true nor false in any world in which $P(M)$ is false. So a common ground c must entail $P(M)$ for $\neg M$ to be acceptable so $P(M)$ is the presupposition of M.

How do we describe the predictions of a trivalent system by heritage rules? Suppose we have a sentence A that is neither true nor false if $P(A)$ is false. We can still state heritage rules for presupposition projection in this theory, it is just that $P(S)$ will be (in principle) trivalent. However, since we are only interested in the cases in which $P(S)$ is true, this is not problematic.[15]

- $P(\neg A) = P(A)$
- $P(A \wedge B) = (P(A) \wedge (A \rightarrow P(B)) \vee (P(B) \wedge (B \rightarrow P(A))$
- $P(A \vee B) = (P(A) \wedge (\neg A \rightarrow P(B)) \vee (P(B) \wedge (\neg B \rightarrow P(A))$

Of course, these rules are exactly the symmetric heritage rules given in section 2.6.

Does the strong-Kleene system yield an answer to questions 1 and 2, above? To the degree that the strong-Kleene truth tables are not just stipulated for each truth-conditional connective but based on some principle it clearly answers questions 1. Since the strong-Kleene truth-tables are obviously unified by a principle of "give truth-values for complexes when they are determinable by the known values of the parts" then the strong-Kleene theory does answer question 1. The system also has an answer for question 2: our language has a strong-Kleene logic and we don't want to assert propositions that are not true or false somewhere in the common ground.[16]

5.2 Symmetric Dynamic Semantics

Rothschild (2008, forthcoming) modifies Heim's dynamic semantics to yield a symmetric and predictive theory of presupposition projection. The basic idea is to treat presuppositional expressions as context change potentials, as in Heim's system, but to take away

[15] As it happens as long as the atomic presuppositions are bivalent the presuppositions of complex expression will inherit this bivalence on the Strong Kleene truth tables, despite the presence of other trivalent formulae in the heritage rules. Since this fact is not material to the discussion here I omit the proof.

[16] Soames (1989) gives an interesting discussion of the conceptual (rather than empirical) difficulties faced by trivalent accounts of presupposition projection, and Beaver and Krahmer (2001) take up some of these issues.

the stipulations on what update procedure can be used for complex expressions. For example, on this system $c + (A \wedge B)$ can be treated either as $(c + A) + B$ or $(c + B) + A$ depending on which is defined. There are two versions of the theory, strict and loose, depending on what the syntactic constraints on update procedures are.

An interesting feature of the strict version of the theory is that its predictions, unlike the two previous theories are unstatable as heritage rules of presuppositions. For on this system the condition put on the common ground for the acceptability of $c + (A \wedge B)$ and $c + (A \vee B)$ is disjunctive. For example, suppose A and B are atomic formulas which are acceptable if $c \models A$ and $c \models B$, respectively. Then, $A \wedge B$ is acceptable in c iff $c \models P(A) \wedge (c + A \rightarrow P(B))$ or $c \models P(B) \wedge (c + B \rightarrow P(A)$. A disjunctive condition of the form $c \models X$ or $c \models Y$ is not equivalent to $c \models Z$ for any Z, and so $P(A \wedge B)$ is not defined by the relation between common grounds and $P()$ spelled out in section 4.1.

A weaker version of the theory, by contrast, is equivalent to the strong-Kleene system, and so has the same heritage rules. Thus, there are at least two distinct routes to deriving the symmetric version of the Karttunen rules in section 2.6 in a way that answers questions 1 and 2 from section 3.

5.3 Schlenker's Transparency Theory and Chemla's Similarity Theory

Schlenker (2006, to appear) develops a nuanced Gricean theory of presupposition projection, which he calls *transparency theory.* Transparency theory depends upon two conversational principles. The first is a principle which states that presuppositional constructions must be *articulated*: in order to say an expression A we need to preface it with $P(A) \wedge$. So instead of just saying *John stopped smoking,* by itself, we must always say instead, *John used to smoke and he stopped smoking.* This principle is obviously untenable in full generality. The second principle, which trumps the first principle, is to avoid redundancy: we should never use any sentence or subsentence S if S is redundant at that point in the discourse.[17] Schlenker spells out these principles very precisely (giving a rather special definition of redundancy), and shows that, combined, these rules predict many of the basic facts of presupposition projection. On one definition of redundancy they yield a symmetric theory, on another an asymmetric theory that exactly matches the standard, asymmetric heritage rules in section 2.4.

There are other accounts that yield the same predictions. Emmanuel Chemla (2008)

[17]The notion of redundancy is motivated by the data discussed in section 2.5.

also gives a theory of presupposition projection which is provably equivalent to the symmetric version of Schlenker's theory in the propositional case. His theory is embedded in a framework which covers two other major topics in formal pragmatics, scalar implicatures and free-choice effects. However, we do not have space to discuss the details of this rich and interesting theory. In addition Schlenker (2008) gives an independent account, his *local context* theory, which also makes the same basic predictions on its symmetric version.

At the end of the day, on Chemla and Schlenker's theories this is the condition for a sentence being acceptable in a common ground, c (in the propositional case): If in some sentence S a presuppositional expression A appears, then S is acceptable in c iff $c \models S_{A/P(A)} \rightarrow S_{A/\top}$ where $S_{X/Y}$ is the sentence obtained by replacing X by Y in S and $\top$ is a tautology.[18]

Despite these complex non-compositional rules it turns out Chemla and Schlenker's symmetric theories yield the following compositional heritage rules for presupposition projection (proofs are in the appendix):

- $P(\neg A) = P(A)$
- $P(A \wedge B) = (A \rightarrow P(B)) \wedge (B \rightarrow P(A))$
- $P(A \vee B) = (\neg A \rightarrow P(B)) \wedge (\neg B \rightarrow P(A))$

5.4 Compositionality of All Theories

All the theories that yield an explicit definition of $P()$ have fully compositional predictions, even if the theories themselves are not stated in a compositional manner. So, in particular, for all A and B and operators $*$, $P(A * B)$ and $P(\neg A)$ depends just on $P(A)$. The strict version of symmetric dynamic semantics also has compositional predictions in the sense that the condition on c for $A * B$ to be acceptable just depends on the condition on c for A to be acceptable and the condition on c for B to be acceptable.

5.5 Asymmetric Variants of All Theories

Given any symmetric theory of presupposition projection, we can easily transform it into an asymmetric one by adding a constraint onto it. One way of doing this is to check

[18] This formulation is due to Chemla (2006), who proves the equivalence of a variety of different rules. See the appendix for more detail.

presuppositions incrementally, an idea developed by Schlenker (2006, to appear). The basic idea is that given any sentence with a linear structure we check presuppositions left to right, making sure at each point that no matter how the sentence ends the entire expression will be acceptable.

Recall that on the symmetric version of these theories some sentence S is acceptable in c iff $c \models P(S)$ where $P(S)$ is the presupposition yielded by the standard heritage rules. Roughly speaking, a sentence is *incrementally acceptable* in a context c if for each starting string α of S, if α is the starting string of any sentence then no presuppositions triggered in α will result in the sentence being unacceptable in c. How exactly to formulate the incremental condition, of course, will depend on the details of the theory in question.[19] The incremental version of Schlenker's theory, the incremental version of the strong-Kleene theory, and the the incremental version of the more constrained version of symmetric dynamic semantics makes exactly the predictions of the standard asymmetric heritage rules in section 2.4.[20]

6 Presuppositions and Logical Equivalence

What is one to make of this profusion of new and revived attempts at explaining presupposition projection? Surely it is premature to endorse any of these theories. But I think there is a significant lesson we can draw. The lesson involves thinking about, generally speaking, what sort of phenomenon presupposition is. One way of thinking about this is to ask at what level are presuppositions invariant. A question you might ask, in this vein, is: if two sentences are logically equivalent, do they have the same presuppositions? The quick answer is *no*. Consider two tautologies, *John stopped smoking or John didn't stop smoking* and *I'm happy or I'm not happy*. The first sentence presupposes that John used to smoke, the second doesn't presuppose anything. Strict logical equivalence is clearly not the right notion for capturing the property over which presuppositions are preserved.

However, presuppositions are preserved across many logical equivalencies. If the arguments in section 2.5 in favor of symmetric theories are correct, then presuppositions

[19] Schlenker (2008) compares different incremental theories, including versions of his own theories and two trivalent systems. An incremental version of the strong-Kleene theory is discussed by George (2008), while an incremental version of the symmetric dynamic semantics is discussed in Rothschild (2008).

[20] Schlenker (2008) shows that the predictions differ when these theories are extended to handle quantifiers.

are invariant across the following logical equivalences.

- $A \wedge B \approx B \wedge A$
- $A \vee B \approx B \wedge A$

The general hypothesis this suggests, which I will call the *Equivalency Hypothesis*, is that presuppositions are preserved over logical equivalences that do not involve the introduction or elimination of tautologies or contradictions. Here are more examples of such equivalencies:

- $\neg\neg A \approx A$
- $\neg(A \wedge B) \approx \neg A \vee \neg B$
- $\neg(A \vee B) \approx \neg A \wedge \neg B$
- $A \wedge A \approx A$
- $(A \wedge B) \wedge C \approx A \wedge (B \wedge C)$

Let us be precise about what it means for presuppositions to be invariant across logical equivalency. The hypothesis can be stated as follows in terms of our compositional presupposition function, $P()$:

Equivalency Hypothesis The function $P()$ from complex sentences to their presuppositions satisfies these conditions:

1. $P(A \wedge B) = P(B \wedge A)$
2. $P(\neg\neg A) = P(A)$
3. $P(\neg(A \vee B)) = P(\neg A \wedge \neg B)$
4. $P(\neg(A \wedge B)) = P(\neg A \vee \neg B)$
5. $P(A \wedge A) = P(A)$
6. $P((A \wedge B) \wedge C) = P(A \wedge (B \wedge C))$

Some notes on the hypothesis. First, it gets more bite if we also assume the compositionally of presuppositions, namely that the presupposition of $A * B$ depends just on the presupposition of A and the presupposition of B. With compositionality in hand, the rules in the equivalence hypothesis can be applied to parts of sentences as well as entire sentences. So, if $P(A) = P(B)$, then $P(X) = P(X_{A/B})$. Second, the hypothesis is obviously limited, since it does not include every non-tautology or contradiction involving logical equivalency. I wanted just to put enough in to the hypothesis in order to illustrate the basic phenomenon, but I leave precise characterization of which logical equivalencies we should concern ourselves with for another occasion. Third, I should note that the hypothesis might be expanded to include a condition which Schlenker argues that adequate theories of presuppositions should satisfy: any two connectives with the same truth-tables and syntax share the same presupposition projection rules. Of course, since I do not discuss such identical connectives this is not entailed by the hypothesis as formulated above, but it is in the spirit of it. It is worth noting that Schlenker's condition alone would not itself entail the Equivalency Hypothesis.

Of course, many obviously false theories of presupposition satisfy the hypothesis: Consider, for example, the theory that for any sentence A, $P(A) = A$. This theory doesn't predict the right presuppositions for simple or complex sentences, but it does satisfy the hypothesis. More interestingly, the so-called *cumulative* theory, usually attributed to Langendoen and Savin (1971), also satisfies the Equivalency Hypothesis. This theory has the following heritage rules:

- $P(\neg A) = P(A)$
- $P(A \vee B) = P(A) \wedge P(B)$
- $P(A \wedge B) = P(A) \wedge P(B)$

This hypothesis, too, makes wrong predictions, even when combined with a cancellation method (as discussed in Soames, 1982).

Despite the ease with which some (wrong) theories satisfy the hypothesis, however, few of the leading theories of presupposition fully satisfy the Equivalency Hypothesis. The symmetrical theories we have discussed do satisfy many parts of the Equivalency Hypothesis. Of course, by virtue of being symmetric, they all satisfy condition 1. Condition 2 is also satisfied by any theory whose heritage rules includes $P(\neg A) = P(A)$, a feature which every viable theory of presuppositions shares, because it is the most basic and easily observable piece of data about presupposition projection. Conditions 3 and 4, De Morgan's law for presupposition projection, also seem to be true. Consider, for instance, this pair of sentences:

(22) a. It's not the case that John didn't used to smoke or that he stopped smoking.

b. John used to smoke and he didn't stop smoking

While (22-a), like many complex examples, is a bit hard to process, it does not seem to presuppose anything when you read *it's not the case* as having wide scope over the whole disjunction. Likewise (22-b) clearly makes no presupposition. It is good, then, that all three of the symmetric theories preserve De Morgan's laws.

This leaves conditions 5 and 6 each of which one of the theories fails to satisfy.

6.1 Symmetric Transparency Violates the Equivalency Hypothesis

First note that Schlenker and Chemla's symmetric theories (which for brevity I'll refer to as *transparency theory*) fails to satisfy condition 5, the rule that $P(A \wedge A) = P(A)$. In fact, on transparency theory $P(A \wedge A)$ is always just $\top$ (on the standard assumption, which Schlenker and Chemla make, that A entails $P(A)$). Given our hypothesis, this should lead to a bad prediction. On transparency theory (23) has no presupposition:

(23) John stopped smoking and John stopped smoking.

Obviously this sentence is independently odd, so perhaps we cannot make judgments about its presuppositions. But, to the degree to which we do have such judgments I think we are likely to say that someone who utters (23) for whatever reason (for example, emphasis of some sort) would normally presuppose that John used to smoke.

This conclusion is supported by the fact that there are close variations on (23) that are not so unusual and are wrongly predicted not to have presuppositions on transparency theory. The critical fact is that in a sentence of the form $A \wedge A$ each the presupposition of each conjunct is filtered by the presence of the other conjunct. The same thing happens when we have two different sentences that happen to have the same presupposition. For instance, (24) is predicted to have no presupposition by transparency theory.

(24) Either John doesn't knows that Mary used to smoke or Mary hasn't stopped smoking.

This sentence is, of course, not of the form $A \wedge A$, but it is instead of the form $\neg A \vee \neg B$ where $P(B) = P(A)$ and, like $\neg A \vee \neg A$ it is predicted by transparency theory to have no presupposition. However it seems to me that an assertion of (24) would naturally presuppose that Mary used to smoke, so the heritage rules of these theories are likely to be incorrect.[21]

6.2 Strict Symmetric Dynamic Semantics Violates the Equivalency Hypothesis

Recall that a feature of the strict version of symmetric dynamics semantics was that heritage rules are unstatable except in terms of conditions on the common grounds. For example $A \wedge B$ is acceptable in common ground c if and only if $c \models P(A) \wedge (A \rightarrow P(B))$ or $c \models P(B) \wedge (B \rightarrow P(A))$. We can, however, reformulate the Equivalency Hypothesis to deal with this more general notion of presupposition. All we need to do is reformulate the rules in terms of a common ground, c, rather than in terms the presupposition function, $P()$. So, for instance, for rule 1 we can write instead $c \models A \wedge B$ iff $c \models B \wedge A$. The rule for conjunction on the strict version of symmetric dynamic semantics preserves commutativity, condition 1, but fails to preserve associativity, condition 6. So it is not true that if $((A \wedge B) \wedge C)$ is acceptable in c then $(A \wedge (B \wedge C))$ is acceptable in c. Consider as an example a case where A, B, and C are atomic formulas such that A has no presupposition, B has the presupposition $P(B)$ and C has the presupposition $P(C)$, and both $A \rightarrow P(C)$ and $C \rightarrow P(B)$ are true. In this case, $((A \wedge B) \wedge C)$ is acceptable in c if and only if $c \models (A \rightarrow P(B))$ or $c \models P(C)$. By contrast $((A \wedge C) \wedge B)$ is acceptable in any common ground. It is a bit hard to test whether or not this result comports with the empirical data, but I suspect it does not. One example of this general form, with the order $C \wedge (A \wedge B)$, is the following:

(25) John stopped running, but he used to run frequently and Mary doesn't know he's stopped.

We could try to test the prediction by forming a question out of (6) or putting it under a wide-scope negation and seeing if the resulting sentence still seems to entail that John used to run or that if he ran he stopped running. However, for obvious reasons we shouldn't expect to have reliable judgments about such complex sentences. Nonetheless, I would think that the prediction that sentences like (25) do have presuppositions is

[21] We might not reject this class of theories on this account, minor modifications could remove this flaw.For example, if you remove the assumption that any expression A entails $P(A)$ then it no longer follows that $P(A \wedge A) = \top$, though the theory still does not satisfy the condition that $P(A \wedge A) = P(A)$.

probably not a very good one.

6.3 Symmetric Karttunen Rules Satisfy the Equivalency Hypothesis

The symmetric Karttunen rules satisfy the Equivalency Hypothesis, which probably explains their relative success. They can be shown to have this property quite easily.[22]

7 Conclusion

This paper is not intended as an argument that natural language has a trivalent logic with the strong-Kleene truth tables or that that natural language uses a symmetric dynamic system. Indeed, both hypotheses seems to me rather improbable. Such systems, of course, explain presupposition projection (in the propositional case) rather well. They do well, I suggest, because they respect the principle that presuppositions are invariant across non-tautology or contradiction involving logical equivalences. There will be other explanations of presupposition and presupposition projection that respect that principle, and we should be interested in looking into these. Nonetheless, if the principle is correct, it will put a sharp constraint on theories of presuppositions generally.

Appendix: Heritage Rules for Symmetric Transparency Theory/Similarity Theory

On Schlenker's theory if sentences A and B are atomic expressions they have a bivalent semantics and entail their encoded presuppositions, $P(A)$ and $P(B)$. Let S be a complex sentence. We will write $X \in S$ to mean that X is an atomic sentence in S. For any such $X \in S$, let $S_{X/Y}$ denote the sentence formed by replacing X with Y in S (for simplicity we will assume that each atomic formula only appears once in any complex sentence S, thus each substitution only has one instance).[23] I will use a definition of the

[22]Schlenker (2008) gives a proof that the presuppositions of the strong-Kleene system are equivalent to a certain supervaluationist system where no sentence letter is repeated. Supervaluationist systems, by construction, preserve all properties over logical equivalence. This leaves only the rule $P(A \wedge A) = P(A)$, and it is trivial to prove that it obtains on the strong-Kleene system since $A \wedge A$ is true or false if and only if A is true or false.

[23]We do not thereby sacrifice any generality since two syntactically distinct atomic sentences can have the same meanings and presuppositions.

acceptability for complex sentences which Chemla (2006) demonstrates is equivalent to Schlenker's slightly more cumbersome definition:

Acceptability of Complex Sentences S is acceptable in C if and only if, for every $X \in S$, $C \models S_{X/P(X)} \leftrightarrow S_{X/\top}$.

Let us define $P(S)$ as the sentence (if it exists) such that S is acceptable in C iff $C \models P(S)$. (It is trivial that this comports with our stipulation of $P()$ for atomic sentences.)

Fact 1. *For any complex sentence A, $P(\neg A) = P(A)$.*

Proof. Given the definition of $P(A)$,

$$\forall X \in A : C \models A_{X/P(X)} \leftrightarrow A_{X/\top}$$

is true if and only if $C \models P(A)$.

It follows immediately that

$$\forall X \in A, C \models \neg A_{X/P(X)} \leftrightarrow \neg A_{X/\top}$$

if and only if $C \models P(A)$ □

Fact 2. *For any complex sentences A and B:*

$$P(A \wedge B) = (A \rightarrow P(B)) \wedge (B \rightarrow P(A))$$

Proof. By the definition of $P(A)$ and $P(B)$, we know that

$$\forall X \in A, C \models A_{X/P(X)} \leftrightarrow A_{X/\top}$$

is true if and only if $C \models P(A)$, and

$$\forall X \in B, C \models B_{X/P(X)} \leftrightarrow B_{X/\top}$$

is true if and only if $C \models P(B)$.

Suppose first that $C \models (A \rightarrow P(B)) \wedge (B \rightarrow P(A))$. Consider an arbitrary $X \in A$. We need to show that

$$C \models (A_{X/P(X)} \wedge B) \leftrightarrow (A_{X/\top} \wedge B)$$

which is equivalent to

$$C \wedge B \models A_{X/P(X)} \leftrightarrow A_{X/\top}.$$

This last statement follows from the assumption that $C \models B \rightarrow P(A)$ and the assumption that

$$\forall X \in A, C \models A_{X/P(X)} \leftrightarrow S_{A/\top}$$

when $C \models P(A)$. By symmetry between A and B, this shows that:

$$(C \models (A \rightarrow P(B)) \wedge (B \rightarrow P(A))) \rightarrow (\forall X \in A \wedge B, C \models (A \wedge B)_{X/(P/X)} \leftrightarrow (A \wedge B)_{X/\top}).$$

Now we need to show the converse of this. Suppose $C \not\models (A \rightarrow P(B)) \wedge (B \rightarrow P(A))$. By symmetry, let us suppose that $C \not\models B \rightarrow P(A)$. We want to show that

$$\forall X \in A \wedge B, C \models (A \wedge B)_{X/P(X)} \leftrightarrow (A \wedge B)_{X/\top}$$

is false. To do this consider the fact that it entails

$$\forall X \in A, C \models (A_{X/P(X)} \wedge B) \leftrightarrow (A_{X/\top} \wedge B)$$

which, as above, is equivalent to:

$$\forall X \in A, C \wedge B \models A_{X/P(X)} \leftrightarrow A_{X/\top}.$$

We know, by assumption, that $C \wedge B \not\models P(A)$. It follows that the statement above is false, from the definition of $P(A)$. □

Fact 3. *For any complex sentences A and B,*

$$P(A \vee B) = (\neg A \rightarrow P(B)) \wedge (\neg B \rightarrow P(A))$$

Proof. The proof is analogous to that of Fact 2, but depends on the fact that $C \models (A \vee B) \leftrightarrow (A' \vee B)$ is equivalent to $C \wedge \neg B \models A \leftrightarrow A'$.[24] □

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[24] I am grateful to Emmanuel Chemla for discussion of these proofs; unfortunately, he is not responsible for any errors.

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