

On the Dynamics of Conversation*

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December 7, 2012

Abstract

On a *static* model of conversational update, there is a mapping from sentences of the relevant language fragment to propositions, and the characteristic discourse effect of successfully asserting a sentence is the addition of the corresponding proposition to the common ground of the conversation. Given the influence of the static picture on much linguistic theorizing, we should like to know: what are the identifying marks of staticness? In this paper we formalize the question and answer it, in the process extending earlier results of van Benthem and Veltman. According to our representation theorem, a fragment of language is static just in case the context change potentials of the fragment exhibit *idempotence* and *commutativity*. This result raises the question how to tell whether a given fragment of natural language exhibits failures of idempotence or commutativity. We clarify this question, and discuss some ways in which putative failures of idempotence and commutativity can and cannot be explained by appeal to pragmatics or to context-sensitivity. In the process we describe some further distinctions within the space of non-static systems, and take a step towards a classification of dynamic systems.

*We are grateful to Chris Barker, Simon Charlow, Emmanuel Chemla, Josh Dever, Thony Gillies, Ben George, John Hawthorne, Benj Hellie, Wesley Holliday, Salvador Mascarenhas, Dilip Ninan, Jim Pryor, Linda Rothschild, Ken Shan, Will Starr, and Malte Willer, and Frank Veltman for discussion. Special thanks to Salah Baouendi, Dana Scott, Robert Stalnaker, Lawrence Valby, and Johan van Benthem for detailed feedback. Thanks also to audiences at Arché, NYU, Gottingen, USC, Rutgers, and Berkeley.

1 Introduction

A familiar picture of conversational dynamics is given by what we can call the *static* picture of communication. On this picture, there is a mapping from sentences of the relevant language fragment to propositions, and the characteristic pragmatic discourse effect of successfully asserting a sentence is the addition of the proposition it expresses to the common ground of the conversation. Given the influence of the static picture on much linguistic theorizing, we should like to know: what are the marks of staticness? That is, what properties indicate whether or not a given a language fragment can be modeled in accord with the static picture? An answer to this question would shed light on what exactly makes a fragment of language “interestingly dynamic” as opposed to static, and would facilitate inquiry into the question whether any given fragment is statically representable.

In this paper we formalize the question of what makes for the staticness of a language fragment and answer it. According to our result, the characterizing feature of staticness is this: the updates to the common ground induced by every sentence of the relevant fragment exhibit *idempotence* and *commutativity*. (We make these notions precise below.) This result raises the question how to tell whether any given natural language fragment exhibits failures of idempotence or commutativity. One might think it would be easy to settle this kind of question, but in fact the matter is often delicate. Below we explain why, and highlight some ways in which putative failures of idempotence and commutativity can and cannot be explained by appeal to pragmatics or to context-sensitivity. In the process we introduce some further distinctions within the space of non-static systems of conversation, and we supply a preliminary classification of dynamic systems. We close by considering the bearing of our results on the large-scale debate about whether a compositional semantics for natural language should take a static or dynamic form.

2 The question

In what follows, we will take it for granted that conversation normally takes place against a *common ground*, a body of information mutually taken for granted, or presupposed, by the discourse participants in context (following, e.g., Stalnaker [1974, 2002], Lewis [1979]). The common ground is the informational context of a conversation. We can think of the common ground of a conversation at a time

as reflecting the state of the conversation at that time. Below we use ‘common ground’, ‘informational context’, and ‘conversational state’ as synonyms. We also take it for granted, following Stalnaker and others, that speech acts can be fruitfully modeled from the perspective of their characteristic effects on the informational context—that is, from the perspective of their characteristic way of changing the state of a conversation. In particular, we will take for granted that every sentence has a *context-change potential*, an operation on informational contexts reflecting the sentence’s conventionally understood characteristic way of changing the state of a conversation.¹

Now the static picture is ordinarily applied in connection with some set of declarative sentences from some fragment of natural language. As we wish to understand it here, it involves the following two assumptions. First, the common ground of any given conversation corresponds to a set of propositions. Second, the characteristic communicative effect of successful assertion (of one of the sentences in the fragment) is the addition of a proposition to the common ground. (It is usual to further assume that the proposition added to the common ground by a sentence is one identical to, or determined by, its compositional semantic value. Strictly speaking our formalization of the static picture will not require this, but for concreteness you may wish to have this kind of idea in mind.)

Natural as they are in many cases, these assumptions are nontrivial modeling ideas at remove from anything like direct observation, and one can easily imagine systems of linguistic communication where things work differently. In particular, one can easily imagine language fragments for which it fails to be the case that every successful update induced by the tokening of a sentence consists simply in the addition of a proposition to the common ground. Indeed, many theorists have argued that various fragments of natural language contain sentences of just this variety. (And even if attention is restricted to declarative sentences.) Notably, a range of *dynamic* approaches to natural language semantics have supplied analyses designed to yield precisely the result that for at least some sentences, the conversational update they

¹Although the term ‘context change potential’ emerged in the dynamic semantics literature, it is important to be clear that one can speak with propriety of the context change potential of a sentence without assuming that the sentence, or the language it is part of, requires a dynamic semantics. To count as an advocate of dynamic semantics, it does not suffice to maintain merely that sentences have context change potentials; rather, one must embrace the further thesis that the context change potential of a sentence is identical to its compositional semantic value. Thus in using the notion of a context-change potential below, we beg no questions on the issue of static versus dynamic semantics.

characteristically induce cannot be reduced to the addition of a proposition to the common ground. The static assumptions are, by contrast, generally associated with so-called *static* approaches to semantics (such as truth-conditional semantics in the style of, e.g., Lewis [1970], Dowty et al. [1980], Heim and Kratzer [1998], etc.). The question whether to theorize about a fragment of language from the perspective of the static assumptions is thus taken to have some bearing on the question whether the best compositional semantics for natural languages will take a dynamic form. (Below we discuss whether this attitude is appropriate.)

To settle whether the static assumptions are appropriate to make in theorizing about any given language fragment, an obvious prior question we should like to answer is the following:

What general properties of a fragment of language indicate whether or not it can, even in principle, be modeled in accord with something like the static assumptions?

That puts the question roughly; we attempt to clarify it in the next section. What we are after is some illuminating independent characterization of the static picture, one that will shed light on its content and facilitate inquiry into whether any given language fragment is statically representable.

3 The question formalized

To address the question, we must clarify what is meant by ‘static’. If we ask, concerning some fragment of language, whether it is statically representable, just what properties of the fragment are relevant to the question? In fact, we need only look to the context change potentials of its sentences. The question whether a language fragment is static, in the sense we are concerned with, is just the question whether its context change potentials can all be construed as proposition-adding operations. Thus if we are interested in understanding whether a fragment is static, we should be able to settle the question given only a formal specification of its context change potentials. A *conversation system* for a language determines such a specification:

Def 1. A CONVERSATION SYSTEM is a triple $\langle L, C, \cdot[\cdot] \rangle$, where L is a set of sentences, C is a set of conversational states, and $\cdot[\cdot]$ is an UPDATE FUNCTION from L to a set of context-change potentials (unary operations) on C (i.e., $\cdot[\cdot] : L \rightarrow (C \rightarrow C)$).²

Whether a language fragment is statically representable is the sort of thing that will be reflected in its conversation system. Note that the above definition presupposes nothing about the structure of conversational states, or about the structure of the language in question, or about the structure of the update function.

Formally, the notion of a conversation system is equivalent to the computational notion of a deterministic labelled state transition system, and can be pictured as a directed graph with labeled arrows. The following illustrates a toy example system:

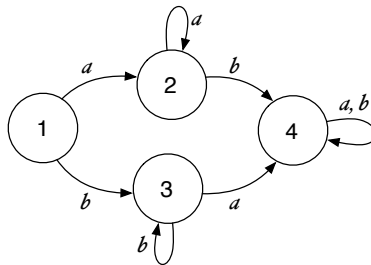


Figure 1: A conversation system with $C = \{1, 2, 3, 4\}$, $L = \{a, b\}$, and $\cdot[\cdot]$ reflected by the labelled arrows.

A conversation system supplies a mapping from sentences to operations on a set. It is also helpful to consider the simpler structure consisting of just the set together with the operations in the image of $\cdot[\cdot]$, abstracting altogether from any particular way of associating the operations with sentences. That is, we can consider a set of context-change potentials themselves, abstracting from any view about which sentences map to which context-change potentials. This would give us what we call a *state system*:

Def 2. A STATE SYSTEM is a pair of a set C of conversational states and a set O of unary operations (context change potentials) o on C (so that $o : C \rightarrow C$ for every $o \in O$).

²The context change potential of a sentence s is $[s]$, and the result of updating a context c with s is $c[s]$ (using postfix notation).

A state system is simply a set together with some operations on the set. Obviously, any conversation system determines a state system: given conversation system $\langle L, C, \cdot[\cdot] \rangle$, the corresponding state system is just the pair $\langle C, O \rangle$, where $o \in O$ iff $o = [s]$ for some $s \in L$. When we speak of the state system *of* a conversation system, or of a conversation system *having* a certain state system, this is the relation we have in mind. The relation between conversation systems and state systems is many-one. The relation *has the same state system as* is an equivalence relation which partitions the space of conversation systems.

States systems may also be pictured as directed graphs with labelled arrows. When they are so pictured, a class of arrows each having the same label should be understood as representing an operation of the state system.³ (Thus in figure 1, for example, we would think of ‘*a*’ and ‘*b*’ as denoting operations on states, not as denoting the sentences of some particular conversation system.)

We said above that to evaluate a language fragment for staticness, we need only look at the context change potentials it makes available. Now we can add that we do not even need to know which sentences of the fragment correspond to which context change potentials. Again, the question is just whether the context change potentials of a fragment can all be construed as proposition-adding operations. We should be able to settle this question given just the set of context change potentials associated with the fragment, and without needing to assume any particular mapping from the sentences of the fragment to context change potentials.

The punchline is this: we may construe the staticness, or non-staticness, of a language fragment as a property of the state system of its conversation system.

Taking this approach, how should the class of static state systems be formally defined? This is equivalent to the question: how should we formally model the idea of a ‘proposition-adding’ operation? Since we are seeking a result with the highest possible level of generality, we are interested in a very abstract formal characterization of this intuitive idea. To clarify and motivate our ultimate definition, it will help to begin with an example of a state system that we think everyone should like to count as static.

Consider the intersective model of assertive conversational update due to Stalnaker [1978]. On this kind of picture, propositions and information contexts are both

³From a graph-theoretic point of view, the difference between a conversation system and a state system is that in a conversation system, a single operation might in principle be represented by more than one class of labelled arrows (i.e., one operation might have several labels). A diagram of such a conversation system might include more structure than a state system can encode.

modeled as subsets of some common domain of points—the set of possible worlds, on the standard model. The propositions which are common ground at a given state of the conversation are those which are true throughout (are supersets of) the informational context. Sentences are semantically associated with propositions, and uttering a sentence is proposing to add the associated proposition to the common ground. To add a proposition to the common ground is to intersect the proposition with the context, the intuitive gloss being that we thereby eliminate from the context the possibilities incompatible with the proposition expressed. On one natural interpretation, the class of state systems where update works in this way may be defined as follows:

Def 3. A state system $\langle C, O \rangle$ is INTERSECTIVE just in case $C \subseteq \mathcal{P}(W)$ for some set W , and there exists some set $P \subseteq \mathcal{P}(W)$ such that for all $o \in O$, there exists $p \in P$ such that for any c , $co = c \cap p$.⁴

We take it as obvious that our ultimate definition of the class of static state systems should count intersective state systems as static.

Now one might stop here, and simply define the static state systems as the intersective state systems. But such a definition would be too limited for a number of reasons.

First, if we were to identify the static systems with the intersective systems, we would be building into staticness structural assumptions that are intuitively irrelevant. The static assumptions we sketched at the outset do not require an unstructured conception of propositions, for example. They say nothing about the nature of propositions. They also do not say that the propositions which are common ground must be closed under some particular operations, though this is required in the context of intersective state systems (where the propositions which are common ground are closed under supersets—under entailment, as standardly understood). Moreover, within an intersective system, adding one proposition to the context ensures that any superset proposition is also incorporated into the context. This means adding one proposition to the common ground is often tantamount to adding many. But nothing in the static picture strictly requires this.

Second and related, we can easily conceive of state systems that are not intersective but which line up with the static assumptions. Suppose for example we assume

⁴We use postfix notation in connection with the operations of a state system, since we are thinking of them as context change potentials. (Thus co is the result of applying o to c ; coo' is the result of applying o' to co ; etc.)

nothing in particular about the structure of propositions, and we think of contexts as sets of propositions. Take conversational update now to be a matter of putting the proposition expressed into the stock of propositions already in the context. That is, “adding a proposition to the context” is just unioning the context with the singleton containing the proposition. The class of state systems where update works in this way can be defined as follows:

Def 4. A state system $\langle C, O \rangle$ is **INCREMENTAL** just in case for some set P , $C \subseteq \mathcal{P}(P)$, and for all $o \in O$, there exists $p \in P$ such that $co = c \cup \{p\}$ for any c .

If one wanted a static conversation system for some fragment of language, but also wanted to model propositions as structured objects, one might reach for an incremental system (or something like it). But obviously, incremental systems are not intersective. Thus we should like to have a definition of ‘static’ broad enough to encompass both intersective and incremental conversation systems.

Third, imagine a state system which is not strictly intersective, but which is nevertheless *isomorphic* to an intersective state system:

Def 5. State systems $\langle C, O \rangle$ and $\langle C', O' \rangle$ are **ISOMORPHIC** just in case there are bijections $f : C \rightarrow C'$ and $g : O \rightarrow O'$ such that $f(co) = f(c)g(o)$ for all $c \in C$ and $o \in O$.

As an example, observe that the state system depicted in figure 1 is isomorphic to the following intersective state system:

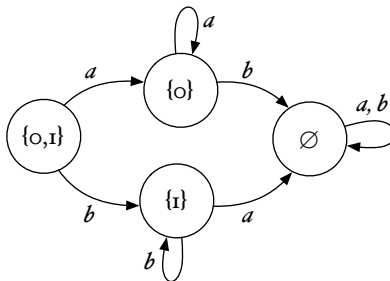


Figure 2: An intersective system with $C = \mathcal{P}(\{0,1\})$ and $O = \{a,b\}$, where a is the operation that which takes a set to its intersection with $\{0\}$ and b is the operation which takes a set to its intersection with $\{1\}$.

The state system depicted by figure 1 corresponds to a labelled directed graph with exactly the same structure as the intersective system in figure 2. From the relevant

abstract point of view, therefore, the two systems manifest the same dynamics. Each could simulate or represent the other. We should therefore want to count as static not just intersective systems, but also any system isomorphic to an intersective system.⁵

Now the question is, what definition of ‘static’ could be general enough to accommodate all of these points? In fact, the following definition will do:

Def 6. A state system is *STATIC* iff it is isomorphic to some intersective state system. (And a conversation system is *STATIC* iff its state system is static.)

This definition avoids the structural assumptions we wanted to avoid. It is also—perhaps surprisingly—general enough to encompass the incremental systems. This is because every incremental state system is in fact isomorphic to some intersective state system. The reverse, however, is not the case.

Fact 1. If a conversation system is incremental, then it is static.

Fact 2. Not every intersective system is isomorphic to some incremental system.

(See the appendix for proofs.) This definition of ‘static’ is highly general. It distills, for our purposes, the relevant structural features of systems which obey the static assumptions.

Equipped finally with this formal definition of staticness, we can restate our target question as one about what makes the state system of a given conversation system static:

What general properties of a state system indicate whether or not it is a static state system?

What we are after is an interesting independent characterization of the class of static state systems. In effect, what we would like is a representation theorem, one which tells us what abstract properties make for isomorphism to intersective state systems. Such a theorem would illuminate the formal character of the static picture. By finding some general properties necessary and sufficient for staticness in the sense defined, we can investigate questions about the staticness or dynamicness

⁵This point applies, *mutatis mutandis*, to incremental systems: given that incremental systems count as static, any system isomorphic to an incremental system should likewise be counted static.

of a natural language fragment by investigating whether the state system of the fragment has or lacks those properties. And we would have an easier time settling whether any given formally specified state system is static.

After reviewing relevant previous results, we supply a representation theorem with the desired character below. But before continuing, we flag two potential points of confusion about our use of the word ‘static’.

First, it is important not to mistake the idea of a static *conversation system* with the idea of a static *compositional semantics*. These notions are at different levels. Unlike a static conversation system, a static compositional semantics says nothing about the context change potentials of the sentences of the language; and unlike a static compositional semantics, a conversation system (static or otherwise) says nothing directly about the compositional semantic structure of the language, as it abstracts from syntactic and subsentential structure altogether.⁶

Second: in the current literature, the jargon of ‘dynamic’ versus ‘static’ is rarely made formally precise. Rather, it usually points to some quasi-technical intuitive distinction among ways of modeling the dynamics of discourse. Now consider the following thesis:

Thesis. The *truly dynamic* conversation systems are the ones whose state systems fail to be static (in the technical sense of ‘static’ just defined).

This is a bit like the Church-Turing thesis: on the lefthand side we have a quasi-technical intuitive notion, and on the right a perfectly clear technical notion. Not unlike the Church-Turing thesis, moreover, this claim can make for idle argument. Our terminological decision to use ‘static’ to refer to a certain specific class of state and conversation systems might give the impression that we embrace this thesis. But we do not—at least, not if this is taken to mean that what we have decided to call ‘static’ reflects the only interesting joint in linguistic nature in this vicinity, or if it is taken to mean we are advancing some conceptual analysis of ‘dynamic’ (whatever that would be). On the contrary, as will become clear below, we think that there is more than one worthwhile distinction among systems to be drawn.

⁶Indeed, the abstract concept of a conversation system does not strictly require that the relevant language have an interesting compositional semantics.

4 van Benthem staticness

Returning now to our target question, we find two results in the literature that approach answering it. The first is due to van Benthem [1986], and is perhaps the most commonly cited observation on this issue.⁷ To explain it, it will be convenient to define the notion of a *van Benthem static* state system:

Def 7. A state system $\langle B, O \rangle$ is VAN BENTHEM STATIC iff there exists a Boolean algebra⁸ B_A , $B_A = \langle B, \wedge, \vee, \neg, \top, \perp \rangle$, such that for all $c \in B$ and $o \in O$,

Eliminativity. $co \vee c = c$

Finite distributivity. $(c \vee c')o = co \vee c'o$

Then we can state the observation as follows:

Fact 3 (van Benthem 1986). If $\langle B, O \rangle$ is a van Benthem static state system with $\langle B, \wedge, \vee, \neg, \top, \perp \rangle$ the associated Boolean algebra, then for all $c \in B$ and $o \in O$: $co = c \wedge \top o$.⁹

⁷See, e.g., Groenendijk and Stokhof [1991b], van Benthem [1996], von Fintel and Gillies [2007], van Eijck and Visser [2010], Muskens et al. [2011].

⁸A BOOLEAN ALGEBRA is a tuple $\langle B, \wedge, \vee, \neg, \top, \perp \rangle$, where B is a set, \wedge, \vee are binary operations on B , \neg is a unary operation on B , and $\top, \perp \in B$, such that: for any $x, y \in B$: (1) $x \vee (x \wedge y) = x$; (2) $x \wedge (x \vee y) = x$; (3) $x \vee \neg x = \top$; (4) $x \wedge \neg x = \perp$.

⁹See van Benthem [1986, p.86], where the point is made for set algebras in the context of a discussion of intersective adjectives.

We note that some authors, such as van Eijck and Visser [2010], cite van Benthem [1989] for Fact 3. That paper, however, presents a distinct claim:

If $\langle C, \wedge, \vee, \neg, \top, \perp \rangle$ is a Boolean Algebra which is idempotent and distributive then “the whole information structure can be represented by a set structure of the ‘eliminative’ kind described earlier” [van Benthem, 1989, p. 38].

It is not entirely clear to us what the statement in quotes means, but from the context it seems that the weakest possible interpretation is as follows: there is a set S and a injective mapping f from C to $\mathcal{P}(S)$ such that for all $c \in C$ and $s \in L$, $f(cs) \subseteq f(c)$. If there is such an f we will say there is an ELIMINATIVE REPRESENTATION of $\langle C, \wedge, \vee, \neg, \top, \perp \rangle$.

The claim is not true. For a counterexample, let $C = \{\emptyset, \{a\}\}$, the Boolean operations have their usual interpretation for the powerset algebra of $\{a\}$, and $L = \{s_1, s_2\}$ such that for all $c \in C$, $c[s_1] = c \setminus \{a\}$ and $c[s_2] = c \cup \{a\}$. s_1 and s_2 are trivially distributive and idempotent, but there is no set-theoretic interpretation of this semantics. It is easy to see that the following three inconsistency properties would be needed for there to be an eliminative representation of this semantics: $f(\{a\}) \neq f(\emptyset)$ (since f is injective), $f(\emptyset) \subseteq f(\{a\})$ (since $\{a\}[s_1] = \emptyset$), and $f(\{a\}) \subseteq f(\emptyset)$ (since $\emptyset[s_2] = \{a\}$). (Note that van Benthem [1989] also mentions what he calls monotonicity in the context in which this proof arises: for all $s \in L, c, c' \in C$: $c \leq c'$ only if $c[s] \leq c'[s]$. Adding this property to the claim does not help however, as the counterexample here also satisfies monotonicity.)

$$\begin{aligned}
\textit{Proof. } c \wedge \top o &= c \wedge (c \vee \neg c)o \\
&= c \wedge (co \vee (\neg c)o) && \text{(Finite distributivity)} \\
&= (c \wedge co) \vee (c \wedge (\neg c)o) \\
&= co \vee \emptyset && \text{(Eliminativity)} \\
&= co
\end{aligned}$$

□

If a state system is van Benthem static, then the update impact of any operation o on c can be factored into two steps: first, let the operation perform its update on the conversational state corresponding to Boolean \top ; second, take the resulting conversational state and output the Boolean meet of it with c . Now the staticness of this kind of state system should be intuitively clear: in a system like this each sentence can be associated with some element in the space of conversational states, and the update of context change potential on any c is equivalent to the Boolean conjunction of that element with c . And indeed, we can establish that any van Benthem static state system is static.

Fact 4. If a state system is van Benthem static, it is static.

(We will prove this claim in §6 below, as a corollary of our main result.) This supplies us with an illuminating, highly general sufficient condition for staticness.

The question now arises about the converse of Fact 4. If a state system is static, does it follow that it is van Benthem static? The answer is negative. This follows trivially from the fact that the set of conversational states in a static state system needn't form a Boolean algebra: consider, for example, any static state system with finitely many conversational states n , such that n does not equal a power of 2.¹⁰

A great many interesting state systems admit of a natural Boolean structure, and in many such cases it will be clear that when evaluated with respect to that structure, the system will be van Benthem static. Still, if a state system fails to be van Benthem static with respect to a particular way of equipping it with Boolean structure, nothing yet follows. To conclude a system is not van Benthem static, we must check every possible way of equipping the system with Boolean structure. And even if we do find that a system is not van Benthem static, it does not follow that it is not static. For again, there will be state systems which simply don't have Boolean structure, and in such cases van Benthem staticness does not usefully apply. These observations lead us to ask whether greater generality can be achieved by assuming less about the structure of the space of conversational states.

¹⁰See figure 3 below for an example of such a state system.

5 Veltman staticness

Veltman [1996] answers this question affirmatively. This brings us to our second result. Rather than assuming Boolean structure for the space of informational contexts, Veltman assumes only that it forms an *information lattice*:

Def 8. A quadruple $\langle V, \top, \wedge, \leq \rangle$ is an INFORMATION LATTICE iff V is a set, $\top \in V$, \wedge is a binary operation on V , and \leq is a partial order on V such that for all $c, c' \in V$:

$$\top \wedge c = c$$

$$c \wedge c = c$$

$$c \wedge c' = c' \wedge c$$

$$(c \wedge c') \wedge c'' = c \wedge (c' \wedge c'')$$

$$c \leq c' \text{ iff there is some } c'' \text{ such that } c \wedge c'' = c'.^{11}$$

Any Boolean algebra determines some information lattice, but not so the reverse. Notably, an information lattice need not include a Boolean \perp , and the cardinality of the space of conversational states needn't be a power of 2. Figure 3 illustrates a simple Veltman static, but not van Benthem static, state system.

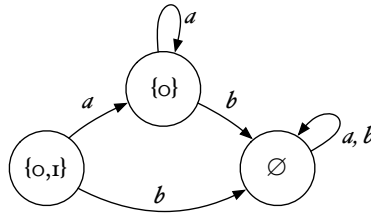


Figure 3: A Veltman static state system that is not van Benthem static. The information lattice is $\langle V = \{\emptyset, \{0\}, \{0, 1\}\}, \top = \{0, 1\}, \cap, \supseteq \rangle$. The state system is $\langle V, \{a, b\} \rangle$.

Using these weaker structural assumptions, we can define Veltman's notion of staticness as follows:

Def 9. A state system $\langle V, O \rangle$ is VELTMAN STATIC iff there exists an information lattice, $V_I, V_I = \langle V, \top, \wedge, \leq \rangle$, such that for all $c, c' \in V$ and $o \in O$,

¹¹The specification of \leq adds no structure as it is induced by \wedge , but we will find the explicit specification convenient below. An intuitive gloss on $c \leq c'$ would be “ c' is at least as informationally strong as c ”.

Idempotence. $coo = co$

Persistence. If $co = c$ and $c \leq c'$ then $c'o = c'$

Strengthening. $c \leq co$

Monotony. If $c \leq c'$ then $co \leq c'o$

Now we observe a result analogous to van Benthem's.

Fact 5 (Veltman 1996). If $\langle V, O \rangle$ is Veltman static state system with $\langle V, \top, \wedge, \leq \rangle$ the corresponding information lattice, then for all $c \in V$ and $o \in O$: $co = c \wedge \top o$.

This result is stated (but not proved) in Veltman [1996]. Here is a proof:

Proof. First we show that $co \leq c \wedge \top o$.

1. $c \leq c \wedge \top o$ (definition of information lattice)
2. $co \leq (c \wedge \top o)o$ (1, Monotony)
3. $\top o = \top oo$ (Idempotence)
4. $\top o \leq c \wedge \top o$ (definition of \leq)
5. $(c \wedge \top o)o = c \wedge \top o$ (3, 4, Persistence)
6. $co \leq c \wedge \top o$ (2,5)

For the other direction, note first in general that if $c \leq c'$, then for any c'' , $c \wedge c'' \leq c' \wedge c''$ (call this 'Order Preservation'). This is because in such a case c' will be equivalent to $c \wedge c'''$ for some c''' , and naturally $c \wedge c'' \leq c \wedge c''' \wedge c''$ (by the definitions of \leq, \wedge).

1. $\top \leq co$ (definition of information lattice)
2. $\top o \leq co$ (1, Monotony, Idempotence)
3. $c \wedge \top o \leq co \wedge c$ (2, Order Preservation)
4. $c \leq co$ (Strengthening)
5. $co \wedge c \leq co \wedge co$ (4, Order Preservation)
6. $co \wedge c \leq co$ (5, definition of \wedge)
7. $co \wedge c = co$ (6, definition of \leq)
8. $c \wedge \top o \leq co$ (3, 7)

□

As with van Benthem staticness, if a state system is Veltman static, then the update impact of any context change potential o on c can be factored into two steps: first, let o perform its update on the informational context corresponding to \top ; second, take the resulting context and output the meet of it with c . And indeed, we can establish the staticness of any Veltman static state system:

Fact 6. If a state system is Veltman static, it is static.

(We will prove this claim in §6 below, as a corollary of our main result.) This supplies us with another illuminating sufficient condition for staticness, and one more general than van Benthem staticness.¹²

As before, the natural next question concerns the converse of Fact 6. If a state system is static, is it Veltman static? The answer is again negative: staticness and Veltman staticness do not coincide. This follows straightforwardly from the fact that the set of conversational states in a static state system need not form an information lattice. To see this, observe for instance that a static state system need not contain an element playing the \top -role.

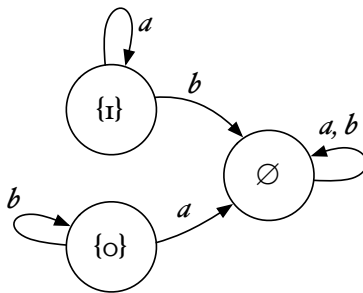


Figure 4: A static state system that is not Veltman static.

Example. Consider the static state system $\langle C, O \rangle$ where $C = \{\{1\}, \{0\}, \emptyset\}$, $O = \{a, b\}$, with $ca = c \cap \{1\}$ and $cb = c \cap \{0\}$ for all $c \in C$. (See figure 4.) Obviously, for every $c \in C$ either $ca = c$ or $cb = c$. Now suppose for contradiction that this system is Veltman static. Then there exists $\top \in C$; hence either $\top a = \top$ or $\top b = \top$. Suppose $\top a = \top$. Then, by Fact 3, for all $c \in C$, $ca = c \wedge \top a = c \wedge \top = c$. But not so, since $\{0\}a \neq \{0\}$. So $\top a \neq \top$. By symmetry $\top b \neq \top$. Hence $\top a \neq \top$ and $\top b \neq \top$. Contradiction.

□

The general thrust of our comments on van Benthem staticness apply *mutatis mutandis* to Veltman staticness. Many interesting state systems admit of a natural

¹²Given Fact 1, it is easy to verify that any van Benthem static system determines an information lattice wherein idempotence, persistence, strengthening, and monotony hold, and hence van Benthem staticness implies Veltman staticness.

information lattice structure, and in many such cases it will be clear that when evaluated with respect to that structure, the system will be Veltman static. However, if a state system fails to be Veltman static with respect to a particular way of equipping it with information lattice structure, nothing yet follows. To conclude a system is not Veltman static, we must check every possible way of equipping the system with information lattice structure. And even if we do find that a system is not Veltman static, it does not follow that it is not static. For again, there are static state systems which simply don't have information lattice structure.

Naturally the ideal, if we could have it, would be to be able state conditions which are both sufficient *and necessary* for staticness in the sense defined, assuming nothing in advance about the structure of the space of conversational states.

6 Staticness characterized

This takes us to the main result of the paper: a representation theorem for staticness. We show that a conversation system is static if and only if it has the properties of *idempotence* and *commutativity*.

Fact 7 (Static representation). A state system $\langle C, O \rangle$ is static iff for all $o, o' \in O$ and $c \in C$,

Idempotence. $coo = co$

Commutativity. $coo' = co'o$

Proof. Clearly, any system isomorphic to an intersective system is idempotent and commutative, since intersection is idempotent and commutative. Hence we need only show that if a state system is idempotent and commutative, then it is static.

Let $\langle C, O \rangle$ be an idempotent and commutative state system. To show $\langle C, O \rangle$ is static, it suffices to produce an injection $h : C \rightarrow \mathcal{P}(C)$ and a function $j : O \rightarrow \mathcal{P}(C)$, such that $h(co) = h(c) \cap j(o)$ for all $c \in C$ and $o \in O$.

Define $j : O \rightarrow \mathcal{P}(C)$ as follows: $j(o) =_{\text{DEF}} \{c \in C : co = c\}$ for all $o \in O$. Thus j takes o to the set of its fixed points.

Define $h : C \rightarrow \mathcal{P}(C)$ as follows: $h(c) = \{c' \in C : cRc'\}$, where cRc' just in case $c = c'$ or c can reach c' from operations in O (i.e., there exist $o_1 \dots o_n \in O$ such that $co_1 \dots o_n = c'$). Obviously, R , the relation of O -reachability, is reflexive and transitive.

Note that from commutativity, it follows that order does not affect update: successive update of c by $o_1 \dots o_n$ is equal to successive update of c by any reordering of $o_1 \dots o_n$, for any $c \in C$ and $o_1 \dots o_n \in O$.

To see that h is an injection, note R is antisymmetric. Suppose cRc' and $c'Rc$, but for contradiction $c \neq c'$. Then $cu_1 \dots u_n = c'$ and $c'v_1 \dots v_m = c$, for some $u_1 \dots u_n, v_1 \dots v_m \in O$. For convenience define U as the functional composition $u_i \circ \dots \circ u_n$ (where $f \circ g =_{\text{DEF}} g(f(x))$); likewise $V = v_1 \circ \dots \circ v_m$. Hence $cU = c'$ and $c'V = c$, and hence $cUV = c$. By commutativity, $cVU = c$. Hence $cVUU = cU$. By idempotence $cVUU = cVU$, so substituting, $cVU = cU$. Substituting again, $c = c'$. Contradiction.

Now if $h(c) = h(c')$, then cRc' and $c'Rc$, since R is reflexive. By antisymmetry, $c = c'$. Hence h is an injection.

Now we show that $h(co) = h(c) \cap j(o)$ for all $c \in C$ and $o \in O$. This equivalent to showing that $\{c' \in C : coRc'\} = \{c' \in C : cRc'\} \cap \{c' \in C : c'o = c'\}$, which is equivalent to showing that for all $c, c' \in C$ and $o \in O : coRc'$ iff cRc' and $c'o = c'$.

Left-right: suppose $coRc'$. There are two possibilities: (i) $co = c'$. Then clearly cRc' . Moreover, by idempotence $coo = co$, so substituting, $c'o = c'$. (ii) For some $u_1 \dots u_n \in O$, $cou_1 \dots u_n = c'$; that is, $coU = c'$. By commutativity, $coU = cUo$. Hence $cUo = c'$. By idempotence $cUo = cUoo$. Substituting, it follows that $c' = c'o$. Moreover, obviously cRc' . Hence cRc' and $c'o = c'$.

Right-left: suppose that cRc' and $c'o = c'$. There are two possibilities. (i) $c = c'$. Then of course $coRc'$. (ii) $cu_1 \dots u_n = c'$ for some operations $u_1 \dots u_n \in O$, i.e., $cU = c'$. Then $cUo = c'o$. Substituting, $cUo = c'$. By commutativity, $coU = c'$. Hence $cRc'o$.

This yields an intersective update system isomorphic to $\langle C, O \rangle$, namely the system $\langle h[C], O' \rangle$, where $h[C]$ is the image of h under C and O' is the set of all operations $o' : h[C] \rightarrow h[C]$ such that for some $o \in O$ and any $c' \in h[C] : c'o' = c' \cap j(o)$.

□

This theorem supplies our desired independent grip on the formal concept of staticness. It also effectively supplies proofs of Fact 4 (If a conversation system is van Benthem static, it is static) and Fact 6 (If a conversation system is Veltman static, it is static). We need only observe that any system which is van Benthem or Veltman static is also idempotent and commutative. Now since any van Benthem static

system is also Veltman static, it suffices to show that any Veltman static system is idempotent and commutative; and since any Veltman static system is idempotent by definition, it suffices to observe that if a conversation system is Veltman static, it is commutative. This point is easy to see: given Fact 5, for any Veltman static system $\langle V, O \rangle$, there is an information lattice V_I , $V_I = \langle V, \top, \wedge, \leq \rangle$ such that for all $c \in V$: $co = c \wedge \top o$. Since \wedge is commutative, for any $c \in V$, $coo' = co'o$.

We can employ this representation theorem as a test for staticness. The test can be applied straightforwardly to artificially specified systems. For example, consider the state system of the conversation system induced by file change semantics (Heim [1982, 1983a]). We can say this system is non-static because it allows for violations of commutativity (e.g., sentence pairs of the form $Fx, \neg Gx$ are not commutative). Dynamic predicate logic (Groenendijk and Stokhof [1991a,b]) and usual versions of update semantics (Veltman [1996]) are non-static because they are neither commutative nor idempotent.¹³ Our result accords with the widely-held view that one of the distinctive features of dynamic semantics is the non-commutativity of conversational update, and of conjunction.¹⁴

We may also use the representation theorem to sharpen the question to what extent natural languages (or relevant fragments thereof) behave in accord with the static assumptions. This can now be understood as the question:

Do the conversation systems appropriate for modeling natural language have state systems respecting idempotence and commutativity?

As we wish to suggest in the next section, answering this question is a subtle matter.

7 Commutativity and idempotence in natural language

Natural language abounds in *prima facie* counterexamples to both idempotence and commutativity. The question is whether the putative counterexamples are bonafide,

¹³Though on update semantics, see §A.2 below.

¹⁴Though note that the fact that the state system of a conversation system is static entails nothing by itself about the semantics of conjunction in the language of the conversation system. Indeed, the language needn't even contain a conjunction operator. (However, if it is assumed, following Stalnaker [1974] and Heim [1983b], that an unembedded conjunction is equivalent to consecutive assertions of the two conjuncts, then staticness will require unembedded conjunctions to be commutative.)

or whether they are instead better explained away along static-compatible lines. In particular cases, this choice is often delicate. To bring that out, we list a number of static-compatible strategies for responding to apparent failures of idempotence and commutativity. Having a basic sense of the range of possibilities for explaining away idempotence/commutativity failure reduces the temptation to use the present theorem to leap too quickly to conclusions. Thus our objective is in this section not to settle the question of static versus non-static, but rather to highlight some considerations relevant for understanding the empirical import of the representation theorem.

7.1 Strategy 1: appeal to pragmatic inappropriateness

A *prima facie* counterexample to idempotence would be any case wherein tokening the sequence ϕ, ϕ differs in acceptability or communicative import from ϕ ; likewise a *prima facie* counterexample to commutativity would be any case where tokening the sequence ϕ, ψ differs in acceptability or communicative import from ψ, ϕ .

One way to explain away such *prima facie* failures is by appeal to facts of pragmatic appropriateness. This is a particularly attractive strategy for covering *prima facie* idempotence failure, at least in many cases. After all, it is pragmatically inappropriate to make overtly redundant discourse moves, and often this will suffice to explain the many cases where tokening the sequence ϕ, ϕ differs in acceptability from ϕ .

The ‘Novelty-Familiarity Condition’ assumed by Heim [1982]—a pragmatic, or any-way non-syntactic, non-semantic constraint on felicity—supplies another illustration of the way a pragmatic principle might be used to block an apparent failure of idempotence. Within her framework, the following two discourses are generally not equivalent in their update effect:

- (1) a. A man walked in.
- b. A man walked in. A man walked in.

The second discourse, but not the first, will normally serve to introduce two discourse referents. But it would be a mistake to conclude that idempotence fails in her system. The Novelty-Familiarity Condition guiding the interpretation of definites and indefinites will preclude the second sentence (1-b) from being readable as equivalent to the first. Specifically, the two indefinites in the discourse will be forced to be read as corresponding to distinct variables in logical form, on pain of

violating the condition. On such a reading, we do not have the same sentence twice over in (1-b), and hence no counterexample to idempotence.

The same kind of story could be applied to commutativity failures. Consider:

- (2) a. [A man]₁ walked in. He₁ was tall.
b. ?He₁ was tall. [A man]₁ walked in.

Heim's condition (or an equivalent pragmatic constraint) could in principle serve to preclude indefinites from coreferring with expressions tokened earlier in the discourse, permitting (2-a) but precluding (2-b). (Obviously, any such constraint would have to be grounded in something other than the conversation system of the language if it is to be construed as a means of preserving staticness.¹⁵)

Consider another case of commutativity failure:

- (3) a. Harry is married. Harry's spouse is a great cook.
b. ?Harry's spouse is a great cook. Harry is married.

What grounds the difference in acceptability? A natural thought is that the first sentence of (3-b) entails the second sentence of that discourse, making that sentence informationally redundant and hence making the discourse pragmatically infelicitous. Thus the pragmatic defect of informational redundancy might again be leaned upon to salvage staticness.

A second (complementary) strategy for example (3) is to appeal to pragmatic constraints on speaker presupposition. On the kind of account developed by Stalnaker [1974], for example, the sentence 'Harry's spouse is a great cook' expresses a proposition which is generally *inappropriate to assert* except in a context where the proposition that Harry is married is already presupposed. In the discourse (3-a), the sentence is tokened relative to such a context (at least assuming that the proposition expressed by first sentence is taken for granted after it is tokened, as it would be the normal course of events); but not so in (3-b). This predicts an the asymmetry in acceptability. This kind of story relies on the following idea:

¹⁵Although we read Heim [1982] as explaining data such as (1) and (2) in a manner strictly compatible with a static conversation system, her full model is of course non-static: the non-commutativity of pairs such as $Fx, \neg Gx$ owes to their context change potentials, and not to extra-semantic constraints like the Novelty-Familiarity Condition.

It is generally inappropriate to say ‘Harry’s spouse is a great cook’ except in a context in which it is part of the mutually presumed background information that Harry is married.

Of course, whether this idea enables one to explain presupposition failure data in a manner compatible with the assumption of a static conversation system for the language depends on whether the appropriateness fact just cited can itself be grounded without appeal to a non-static conversation system.

7.2 Strategy 2: appeal to semantic context-sensitivity

Numerous cases of superficial idempotence failure seem not to trade on pragmatic appropriateness, but rather on semantic context-sensitivity. Indexicals make for obvious examples. Consider the discourses:

- (4) a. *Speaker A*: I love you.
b. *Speaker B*: I love you.
- (5) a. This [*pointing to the lamp*] is old.
b. This [*pointing to the table*] is old.

In each of these cases that the relevant (b.)-sentence obviously serves to add new information to the common ground. It is clear what the static-friendly rejoinder to these examples will be, in general terms. It will be that although every sentence characteristically serves to add a proposition to the common ground along familiar static lines, the propositions expressed by the respective (b.)-sentences are different from the propositions expressed by the respective (a.)-sentences. In general, what proposition is expressed is a function of context (‘context’ understood broadly in the sense of Kaplan [1977/1989] and Lewis [1980], roughly as the centered world locating the speaker at a given time in the discourse). Virtually everyone who conceives of assertion as in part a matter of expressing propositions believes that in at least some cases, the proposition determined is a function of the context. This has been the standard view about sentences containing indexicals and demonstratives since at least Kaplan [1977/1989].

From a conversation systems perspective, context-sensitivity can be understood as the idea that the ‘language’ of the conversation system is given, not by a set of

sentences, but rather by a set of sentence-context pairs. Equivalently, if we picture the conversation system as a state transition system, it is the idea that the labels for the transitions of the system are given by sentence-context pairs. See figure 5 for a simple illustration of the idea in connection with example (4).

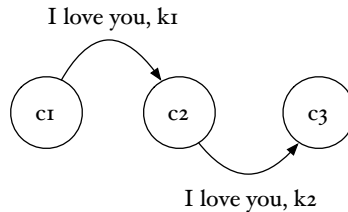


Figure 5: An idempotence-compatible fragment of a conversation system for example (4). k_1 and k_2 are worlds centered on distinct speakers and times (i.e., distinct contexts).

Idempotence does not require state c_2 to equal c_3 , because the arrow connecting c_2 and c_3 does not share a common label with the arrow connecting c_1 and c_2 . The basic move here is as simple and as it is powerful: any *prima facie* counterexample to idempotence might in principle be blamed in this way on tacit context-sensitivity instead. The present point illustrates the way in which the notion of a ‘language’ appropriate to the conversation systems perspective may be forced to depart from the more usual one.¹⁶

The same idea for preserving idempotence can be applied to example (5). It can also be applied in connection with apparent commutativity failures involving anaphora. Consider again:

- (2) a. [A man]₁ walked in. He₁ was tall.
 b. ?He₁ was tall. [A man]₁ walked in.

On the best known static-friendly approaches to anaphora, the reference of anaphoric pronouns is understood to be mediated by appeal to a contextually salient description, one sensitive in some way to the preceding discourse. The literature contains a range of subtly different developments of this idea (see among others Evans [1977a,b], Cooper [1979], Kaplan [1977/1989], Heim [1990], Neale [1990], Heim and Kratzer

¹⁶To the extent that this kind of brute appeal to a richer language is required, it may limit the interest of a conversation systems perspective. But insofar as context-sensitivity can be restricted to what we (below) call *information-sensitivity*, a conversation systems approach can model context-sensitivity without adding structure to the language in question.

[1998], Stalnaker [1998], Elbourne [2005]). But abstracting away from many details of implementation, we can say that on most of these approaches, putative counterexamples to commutativity such as (2) are explained away in part by appeal to the thought that the sentence ‘He was tall’ does not generally express the same proposition across (2-a) and (2-b), owing to the context-sensitivity of the pronoun.

To what aspect of the context is the pronoun in (2-a) supposed to be sensitive? On most of these views, part of what fixes the semantic contribution of the pronoun is the very fact that the first sentence (‘A man walked in’) was uttered. That is, the semantic contribution of the pronoun is sensitive to a fact about the history of the discourse itself. That history is manifestly different across (2-a) and (2-b). These accounts lean on the simple fact that in making a successful assertion, one not only updates the state of the conversation; one also fixes a fact about the history of the discourse, merely by talking. Thereby one change the context (in the Kaplan [1977/1989]/Lewis [1980] sense of ‘context’) in a manner relevant for the interpretation of expressions in subsequent utterances.

7.3 Strategy 3: appeal to the flux of the common ground

There is third way many of the above examples can be argued not to impugn stat-icness. This is by appeal to the simple fact that in anything like normal discourse, the common ground which results from the update of a sentence in that discourse is never identical to the common ground that is updated by the subsequent sentence in that discourse.

This is particularly obvious in the case of (5-a) and (5-b). There an event of ostension occurs in between sentence tokenings, and this event will become common ground as soon as it happens in any normal conversation. As a result, the update associated with (5-b) will apply to something other than the output of the update associated with (5-a). And this entails that this discourse is not—at least without further argument—an example of a failure of idempotence in the strict sense.

Other examples can be explained in the same fashion. Suppose a sergeant is inspecting a cadet’s uniform. He shouts, “Turn around!” The cadet obliges. The sergeant, satisfied that the cadet’s shirt is properly tucked, again shouts “Turn around!” The cadet then returns to his original position. Here it is clear that the sergeant’s second command is not redundant, and nor is it merely adding emphasis to the first command. Nevertheless, plausibly this involves no failure of idempotence, as the second

command does not update the conversational state resulting from the update due to the first command. Rather, it updates a conversational state which incorporates (*inter alia*) the information that the first command was satisfied. If the cadet had not obliged by turning around after the first command, the sergeant’s full discourse (“Turn around! Turn around!”) would have amounted to one (emphasized) directive to turn 180 degrees, not a directive to turn 360 degrees. Thus the example is not a counterexample to idempotence.

The general point here has very wide application. As Stalnaker [1998] has emphasized, uttering a sentence does not only change the common ground in virtue of the context change potential associated with the sentence by the semantics and pragmatics of the language. It also changes the common ground by adding to it the very fact that the utterance was made. Even if I don’t accept your assertion, we will go on to presuppose that you uttered a certain sentence at a certain time, in an effort to add something to the common ground. The fact that assertions change the context in this secondary way makes it much less straightforward than it may seem to probe idempotence and commutativity in natural language. In theorizing from a conversation systems perspective, we generally abstract from the secondary kind of change to the context that assertions make, the change induced by the fact of the utterance itself. But there is no escape from this secondary kind of change in ordinary communication. As a consequence, merely reversing the order in which sentences are uttered does not generally result in commutation in the strict sense. Likewise, merely repeating a sentence does not yield a case where the context change potential of a sentence applies to its value relative to an initial conversational state. Figure 6 illustrates the basic difficulty with respect to commutativity.

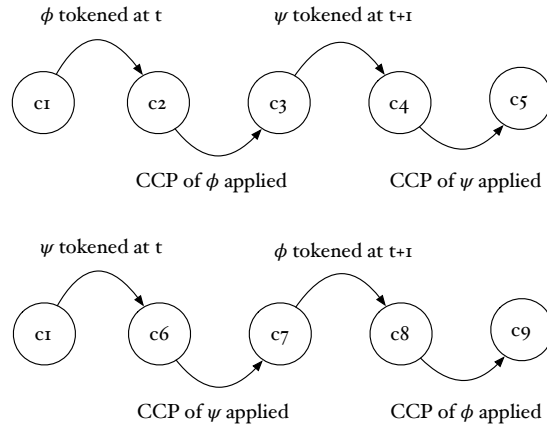


Figure 6: Merely reversing the order of sentences in ordinary natural language conversation does not result in commutation. Commutativity does not require that $c_5 = c_9$.

One might have thought that if a conversation system containing ϕ and ψ is commutative, it would follow that tokening ϕ , and then ψ , against a conversational state c_1 would have to result in the same conversational state as would result from tokening ψ , and then ϕ against c_1 . But not so. In real examples, there is no way to apply two context change potentials in immediate succession, for updates owing to the mere facts of utterance will always intercede. But commutativity is a claim about the equivalence of two ways of applying a pair of context change potentials in immediate succession. Thus commutativity is not the sort of thing we can just observe (or fail to observe) in ordinary examples. The point applies *mutatis mutandis* to idempotence.

This highlights the extent to which these properties, while much closer to the linguistic surface than the abstract terms in which the notion of a static conversation system is defined, are still at some nontrivial remove from observation. This does not mean that it is impossible to argue against commutativity or idempotence for natural language conversation systems. It means that one always has to argue that one's counterexample really is a counterexample, as opposed to something to be chalked up to the ever-present secondary effects on the conversational state induced by the relevant speech acts. To say any given case supports or undermines commutativity or idempotence is in part to make a substantive judgment call about the role of these secondary effects on conversational update.

7.4 Information-sensitivity

A final strategy for preserving (aspects of) the static picture in the face of recalcitrant data might be thought of as a special case of the second strategy. This is the idea of treating the proposition expressed by a sentence as a function of a specific feature of the context: the common ground. Lewis [1979] is readable as defending this kind of context-sensitivity for a wide variety of expressions. Heim [1982] can be read as taking this approach for sentences containing presupposition triggers. Stalnaker [1998] suggests this kind of approach for anaphora, and much earlier defended it for conditionals (Stalnaker [1975]). The update semantics for epistemic modals developed by Veltman [1996] can be reconstrued as encoding the idea that epistemic modal sentences express propositions which are a function of the input conversational state. The same is true for the semantics for epistemic modals and conditionals in Yalcin [2007]. Klinedinst and Rothschild [2010] appeal to information-sensitivity to model certain interactions between modals and connectives. Many other applications of the idea of information-sensitivity can be found in the literature.

Information-sensitivity is worth distinguishing from other kinds of context-sensitivity, because in order to model it from a conversation systems perspective, there is no need to think of the labels of the system as sentence-context pairs, as other kinds of context-sensitivity seem to require. By design, conversation systems encode the facts about what information is mutually presupposed prior to update, so they already have the resources to represent the way in which the proposition expressed by a sentence might depend on the input conversational state.

The information-sensitive picture has a strong affinity with the static picture, as on both approaches, update can be construed as proposition-adding. But information-sensitivity is not rightly conceived of as a special case of the static picture. On the contrary, as we explain in the next section, the reverse is the case: the static systems are best understood as a special (limiting) case of the information-sensitive systems. Moreover, interestingly information-sensitive systems are generally not static: even if update is always a matter of adding a proposition to the common ground, a system will not generally be static if the proposition added can vary with the input conversational state. Thus while appeal to information-sensitivity might enable one to preserve the idea that update is always proposition-adding, it is not a strategy for explaining away apparent counterexamples to idempotence or commutativity.

8 Information-sensitivity characterized

To clarify this point, and the general relationship between information-sensitivity and staticness, we should like to characterize precisely the class of conversation systems where (i) each sentence serves to add a proposition to the common ground, as on the static assumptions, but (ii) at least some times, the proposition expressed is a function of the informational context. Call such conversation systems *information-sensitive*. Just as we defined the static conversation systems as those having a static state system, and picked out the static state systems as those isomorphic to some intersective system, we may likewise define the class of information-sensitive conversation systems as those whose state systems are isomorphic to a certain kind of concrete system. The kind of concrete system we need is one we have met before. We need an *eliminative* system:

Def 10. A state system $\langle C, O \rangle$ is **ELIMINATIVE** just in case $C \subseteq \mathcal{P}(W)$ for some set W , and there exists some set $P \subseteq \mathcal{P}(W)$ such that for all $o \in O$ and $c \in C$, there exists $p \in P$ such that $co = c \cap p$.

Recall we encountered this property (in a different guise) in the definition of the van Benthem static systems. The above definition is intended to facilitate comparison to the intersective systems.¹⁷ This says that for any choice of conversational state and update operation, the action of updating that state with that operation is equivalent to the action of intersecting that state with some set. Note the definition does not require that a given operation always be equivalent to intersection with some particular set. This definition in turn gives us our desired concept of information-sensitivity:

Def 11. A state system is **INFORMATION-SENSITIVE** iff it is isomorphic to a eliminative system. (A conversation system is **INFORMATION-SENSITIVE** iff its state system is information-sensitive.)

It should be clear that any intersective system is eliminative. The static state systems are a limiting case of the information-sensitive systems. (And the static conversation systems are a limiting case of the information-sensitive conversation systems.) Information-sensitivity is a generalization of the notion of staticness. We could say that a state/conversation system is *interestingly* information-sensitive

¹⁷A simpler but equivalent definition would be: a state system $\langle C, O \rangle$ is **ELIMINATIVE** just in case $C \subseteq \mathcal{P}(W)$ for some set W , and for all $c \in C, o \in O, co \subseteq c$.

iff it is information-sensitive and not static. For an illustration of a interestingly information-sensitive system, see Figure 7.

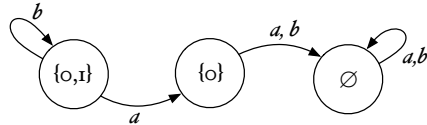


Figure 7: A non-static information-sensitive state system. The system is antisymmetric (as defined below) but is neither commutative nor idempotent.

We may seek formal properties independently characterizing the class of information-sensitive systems, just as we sought formal properties characterizing the static systems. One simple formal property information-sensitive systems clearly have is that they prohibit backpedaling: once the conversation moves beyond a given conversational state, it cannot return to that state by any series of operations. We could say that these systems abide by their updates. Formally:

Def 12. A state system $\langle C, O \rangle$ is **ANTISYMMETRIC** iff for all $c, c' \in C$, if c is O -reachable from c' and c' is O -reachable from c , $c = c'$.

From a graph-theoretic point of view, a state system is antisymmetric just in case every cycle in the graph is a loop (i.e., a cycle of length 1, beginning and ending at the same point by tracing a single arrow). If a system is not antisymmetric, it enables looping between two or more conversational states.

Now we can observe antisymmetry is both necessary and sufficient for information-sensitivity:

Fact 8. A state system is information-sensitive just in case it is antisymmetric.

Proof. Left-to-right: any system isomorphic to a quasi-intersective system is antisymmetric, since in such systems update can only map a conversational state to itself or to a subset of itself.

Right-to-left: suppose $\langle C, O \rangle$ is antisymmetric. To show $\langle C, O \rangle$ is information-sensitive, it suffices to produce an injection $h : C \rightarrow \mathcal{P}(C)$ and a function $i : (O \times C) \rightarrow \mathcal{P}(C)$, such that $h(co) = h(c) \cap i(o, c)$ for all $c \in C$ and $o \in O$.

Let $h(c) =_{\text{DEF}} \{c' \in C : cRc'\}$. Let $i(o, c) =_{\text{DEF}} h(co)$. Obviously R is reflexive, and since $\langle C, O \rangle$ is loopless it is antisymmetric. Hence h is an injection. Now if $coRc'$, then clearly $cR'c$. Hence $h(co) \subseteq h(c)$. Hence $h(co) = h(c) \cap h(co)$. Hence $h(co) = h(c) \cap i(o, c)$.

□

The class of state (conversation) systems that are information-sensitive is quite broad. As we note below, it includes many dynamic semantic systems in the literature. When thinking about language from a state (conversation) systems perspective, the antisymmetry property provides one very natural further boundary outside the class of static systems.

9 Some levels of dynamicness

Given the distinctions marked in logical space by staticness and by information-sensitivity, it is natural to ask whether there are further interesting distinctions to be drawn in between them. The question merits separate detailed investigation, but as a preliminary, we can display the venn diagram of the properties so far discussed, and name some relevant regions of logical space. See figure 8.

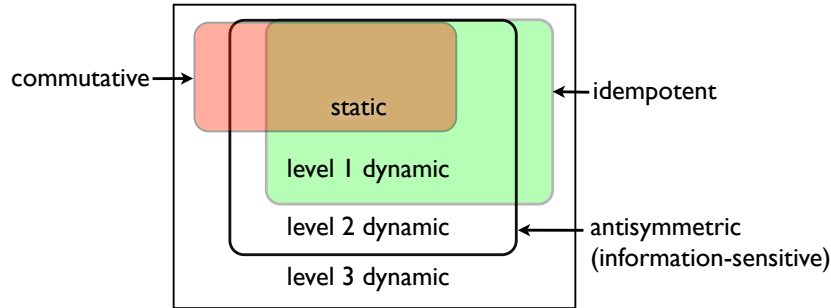


Figure 8: Logical relations between properties of state systems, and some varieties of dynamic systems.

Loss of commutativity seems to be the common thread among dynamic semantic systems discussed in the literature (indeed, we are unaware of nonstatic dynamic semantic frameworks which are commutative). Setting such systems aside, and with an eye towards concrete systems already existing in the literature, one natural

ordering begins with staticness and then progressively subtracts the properties of commutativity, idempotence, and antisymmetry. This would make for three ‘levels’ of dynamic systems. At the first level we have the noncommutative idempotent antisymmetric systems. Heim’s file change semantics is an example of a system at this level. At the next level we have the noncommutative nonidempotent antisymmetric systems. Many dynamic systems in the literature reside at this level: for example, the extensions of Veltman’s update semantics found in Beaver [2001], Gillies [2004]; the system given in §3 of Groenendijk et al. [1995]; the system ABLE (Beaver [2001]); the semantics for counterfactuals in Gillies [2007]; the systems for modals and conditionals in Willer [2010a], Willer [2010b]; the dynamic probabilistic semantics described in Yalcin [2012]. Finally, we could give up all three of commutativity, idempotence, and antisymmetry, adopting a system not even representable as information-sensitive. Dynamic predicate logic is a clear example of such a system (for relevant discussion, see Groenendijk and Stokhof [1991b]).¹⁸

Of course, this is merely one way of beginning to carve up the logical space of systems. Presumably there are further interesting distinctions within what we are calling the second level; and there may be other interesting distinctions crosscutting our classification altogether.¹⁹ For some discussion of adding the property of persistence into the classificational mix, see the appendix.

10 Closing: what has any of this to do with semantics?

We have pointed out some of the subtleties involved in determining what kind of conversation system a fragment of language might have. Now let us ask what follows about the semantics of a language fragment, given facts about its conversation system. Suppose that the conversation system appropriate to some natural language fragment were non-static. What if anything would follow concerning the character

¹⁸Thanks here to Malte Willer for discussion.

¹⁹Groenendijk and Stokhof [1991b] note that dynamic predicate logic is distributive but not eliminative (antisymmetric), whereas update semantics is eliminative (antisymmetric) but not distributive. If one takes distributivity as an important mark of staticness—as would seem natural in light of Fact 3 above—then in this respect it would be a property that crosscuts our classification, and moreover one that might encourage locating DPL and update semantics at roughly equivalent “level of dynamicness”. We are disinclined to view distributivity *per se* as a step in the direction of staticness in the absence of the assumption of antisymmetry. Moreover, it is not clear to us that the relevant abstract property (of isomorphism to a distributive system) is a particularly natural property, on par with the others we are discussing. But the question deserves further discussion.

of the compositional semantics for that fragment?

We might distinguish two cases, depending on whether the system is antisymmetric. Suppose it is antisymmetric (i.e., it is interestingly information-sensitive). Then, as far as we can see, not much follows. Certainly, it would not follow that the compositional semantics of the fragment must take the form of a typical dynamic semantics, assigning context change potentials as the compositional semantic values of the sentences. It could (for instance) take the form of an intensional semantics mapping sentences to functions from world-informational context pairs to truth values—that is, it could take the form of a static, but information-sensitive, semantics, one associating sentences with propositions relative to conversational states. (Compare Yalcin [2007], Kolodny and MacFarlane [2010], Klinedinst and Rothschild [2012].)²⁰ Or more mundanely, if conversational states were represented with a structure rich enough to track order of update—say, with a structure determining an ordered sequence of propositions, with assertion understood as *inter alia* adding the proposition asserted to the end of the sequence—the corresponding conversation system would be nonstatic. But obviously, the semantics of the language for such a conversation system could easily be a classical ‘static’ truth-conditional semantics. Thus the fact that the conversation system for a language is interestingly information-sensitive does not *per se* entail that its compositional semantics must be dynamic. Rather, what we can say is that in particular cases, interesting information-sensitivity might be marshalled, together with other considerations, to make a defeasible case for a dynamic semantics.

Now suppose that the system for the fragment in question lacks antisymmetry—it is at our third level of dynamicness. Here too, it does not immediately follow that the semantics of the language must take the form of a typical dynamic semantics. After all, it is not hard to construct artificial non-antisymmetric systems which would nevertheless dovetail with a static semantics. (One might, for example, build it into the pragmatics of the language that the expression of some particular proposition \perp has the dynamic role of backpedaling the conversation one step.) Again, what we can say is that in particular cases, the non-staticness of the conversation system might be marshalled, together with other considerations, to make a defeasible case

²⁰One might object that in such systems, sentential semantic values are identified with functions from conversational states to things of the type of conversational states; hence are effectively semantically associated with context change potentials; hence are dynamic semantic systems. But the intensions associated with sentences on these accounts are not equivalent to the context change potentials of those sentences. More on the question what makes a compositional semantics “really dynamic” below.

for a dynamic semantics.

We could also ask what, if anything, follows about compositional semantics from the staticness of the language’s conversation system. Here is it natural to think that staticness entails that—at least insofar as the language has a compositional semantics at all—it has a semantics along traditional, static lines. Is there a reason to doubt this? Could the conversation system of a language be static, simultaneous with its compositional semantics being robustly dynamic? Perhaps there could be robust intrasentential dynamics not visible at the level of intersentential dynamics?

To clarify this question, we require some abstract and principled formal specification of what makes a compositional semantics interestingly dynamic (or static). We do not know what such a principled formal specification would look like, however. Obviously, it would not suffice to define ‘dynamic semantics’ as a semantics wherein the semantic values of sentences are context change potentials. It is a familiar point that there are ways of reformulating paradigm examples of static semantic theories in such a dynamic fashion while preserving all entailment and consistency relations between sentences.²¹ Such reformulations are most naturally described as static at heart, and only superficially dynamic. The dynamicness of a formal semantics, we take it, is not worn on its sleeve; it is some more abstract property. The problem is to say what this abstract property is. Finding an adequate statement of this property seems to us a highly nontrivial matter, one interacting with a number of complex issues at the syntax/semantics interface. There are probably many interesting definitions to be found, but going into them is far beyond the scope of this paper. Suffice to say that we have not attempted to address questions of intrasentential dynamics here. Our inquiry has been at the conversation systems level, at the level of discourse update. Our results might be thought a useful preliminary to questions of intrasentential dynamics.

While the feature of dynamic semantics often emphasized is its identification of sentential semantic values with context-change potentials, dynamic semantic systems usually incorporate features which are independent of this identification but which lead to much of their power. The system of Heim [1982], for instance, includes unselective quantification (drawing on Lewis [1975]), the uniform treatment of pronouns and indefinites as variables, and the incorporation of discourse referents into the representation of context (drawing on Karttunen [1976]). These ideas do not fundamentally require the identification of sentential semantic values with context-

²¹For some relevant discussion, see von Stechow and Gillies [2007].

change potentials in order to be implemented.²² We think it might prove worthwhile to explore non-dynamic semantic systems incorporating these ideas, if only to gain further perspective on dynamic semantics.

A Appendix

A.1 Static and incremental

We demonstrate that the class of incremental systems is a proper subset of the class of static systems.

Def 13. A state system $\langle C, O \rangle$ is **INCREMENTAL** just in case for some set P , $C \subseteq \mathcal{P}(P)$, and for all $o \in O$, there exists $p \in P$ such that $co = c \cup \{p\}$ for any c .

Fact 1. If a conversation system is incremental, then it is static.

Proof. Suppose $\langle C, O \rangle$ is an incremental conversation system. Then for some set P , $C \subseteq \mathcal{P}(P)$, and for all $o \in O$, there exists $p \in P$ such that $co = c \cup \{p\}$ for any c . Define $\llbracket \cdot \rrbracket : O \rightarrow P$ such that for all $o \in O$: $\llbracket o \rrbracket =_{\text{DEF}}$ the p such that for all c , $co = c \cup p$. Define $j : O \rightarrow \mathcal{P}(P)$ as follows: $j(o) =_{\text{DEF}} P \setminus \{\llbracket o \rrbracket\}$ ($= \{\llbracket o \rrbracket\}^c$). Define $h : C \rightarrow \mathcal{P}(P)$ as follows: $h(c) =_{\text{DEF}} c^c$. Clearly h is an injection.

Now it remains to show that $h(co) = h(c) \cap j(o)$ for all $c \in C$ and $o \in O$. Since $\langle C, O \rangle$ is incremental, $co = c \cup \{\llbracket o \rrbracket\}$. Taking the complement of each side, $(co)^c = (c \cup \{\llbracket o \rrbracket\})^c$. Distributing on the right, $(co)^c = c^c \cap \{\llbracket o \rrbracket\}^c$. Hence $h(co) = h(c) \cap j(o)$. □

Fact 2. Not every intersective system is isomorphic to some incremental system.

Proof. Consider an intersective conversation system $\langle C, O \rangle$ such that: (i) $c(o \wedge o')o = c(o \wedge o')$; (ii) $c \neq co \neq c(o \wedge o')$. Suppose for contradiction the system is isomorphic to an incremental system. Then there exists a bijection h and a function j into singletons such that for all $c \in C, o \in O, h(c) \cup j(o) = h(co)$. Given such a mapping, it follows that

$$h(c(o \wedge o')o) = h(c) \cup j(o \wedge o') \cup j(o)$$

²²Indeed, Schlenker [2009] has argued that even the notion of a local context can be reconstructed in a static setting, and so is separable from the assumption that sentential semantic values are context-change potentials.

From this and (i), it follows that

$$h(c) \cup j(o \wedge o') \cup j(o) = h(c) \cup j(o \wedge o')$$

Hence $j(o) \subseteq h(c)$ or $j(o) \subseteq j(o \wedge o')$. Suppose $j(o) \subseteq h(c)$. Then $h(c) = h(c) \cup j(o) = h(co)$. Since h is a bijection, $c = co$, contradicting (ii). So suppose instead $j(o) \subseteq j(o \wedge o')$. Since j is into singletons, it follows that $j(o) = j(o \wedge o')$. Hence $h(co) = h(c(o \wedge o'))$. Therefore $co = c(o \wedge o')$, contradicting (ii). □

A.2 Adding persistence

Consider the property of *persistence*:

Def 14. A state system $\langle C, O \rangle$ is PERSISTENT iff for all $c, c' \in C$, if $co = c$ and cRc' , then $c'o = c'$.

Persistence says that anything reachable by c is a fixed point of whatever c is fixed point of. Persistence is entailed by commutativity, and is logically independent from idempotence. However, if antisymmetry is assumed and operations are closed under functional composition, idempotence entails persistence.

Proof. Let $\langle C, O \rangle$ be antisymmetric and idempotent, with O is closed under functional composition. Suppose $co_1 = c$ and c' is O -reachable from c . Then since O is closed under functional composition, there is some operation o_2 such that $co_2 = c'$, and moreover there is some operation $o_3 = o_1 \circ o_2$. Now $co_3 = c'$. By idempotence, $co_3o_3 = c'$. Hence substituting, $c'o_3 = c'$, meaning $c'o_1o_2 = c'$. Hence c' is reachable from $c'o_1$. Since obviously $c'o_1$ is reachable from c' , it follows from antisymmetry that $c'o_1 = c'$. Hence $\langle C, O \rangle$ is persistent. □

(This point is worth noting, since dynamic systems containing something equivalent to the Stalnaker-Heim dynamic conjunction operator—where for all $c \in C, s \in L$, $c[s \wedge s'] = c[s][s']$ —will have state systems whose O is closed under functional composition.)

Moreover, idempotence and persistence together entail antisymmetry.

Proof. Let $\langle C, O \rangle$ be persistent and idempotent. Let $c, c' \in C$ be such that $cRc', c'Rc$. Hence $cu_1 \dots u_n = c', c'v_1 \dots v_m = c$, for some $u_1 \dots u_n, v_1 \dots v_m \in O$. By idempotence, $c' = c'u_n$. By persistence, c' is a fixed point of anything c is a fixed point of, and vice versa, hence $c = cu_n$. By persistence, $cu_1 \dots u_{n-1}$ is a fixed point of u_n . Hence $cu_1 \dots u_{n-1} = c'$. Repeat until $c = c'$. Hence $\langle C, O \rangle$ is antisymmetric.

□

Thus if we add persistence to figure 8, we get figure 9:

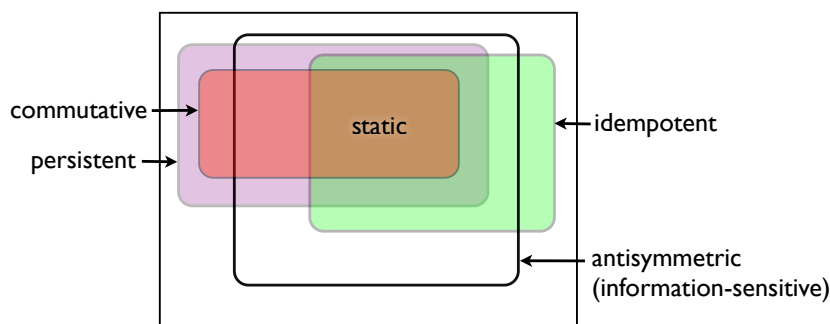


Figure 9: More logical relations between properties of state systems.

In standard applications, idempotence and persistent tend to stand or fall together. Thus file change semantics is both idempotent and persistent, whereas all of the level 2 dynamic systems mentioned above are neither idempotent nor persistent. If there were some interesting antisymmetric systems having one but not both of these properties, we would have some motivation for complicating the initial classifications of dynamic systems mentioned above.²³ Likewise, if there were some interesting non-antisymmetric but idempotent or persistent systems, that would motivate further categories.

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²³In the formulation of update semantics given in Veltman [1996], *might* can only take wide-scope, and the resulting system is idempotent but nonpersistent. But it is usual to extend this system to allow arbitrary embedding of epistemic modals; and on the standard extensions, idempotence is lost (Groenendijk et al. [1995]). (Thanks here to Thony Gillies for discussion.)

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