

CHANGING BELIEFS

HANDOUT 3

NILANJAN DAS

UCL

o. A Problem for Bayesians

- *Two Constraints.*

- **HOLISM.** For any piece of evidence E and any proposition P , there is a 'defeater' proposition D such that an agent's credence in P upon receiving evidence E should depend on her credence in D .
- **COMMUTATIVITY OF EVIDENCE.** In any case where an agent receives two pieces of (non-self-locating) evidence E_1 and E_2 , historical facts about the order in which these pieces of evidence are acquired make no difference to what is rational for the agent to believe.

- *A Problem.*

WEISBERG'S CLAIM. If perception can be a source of evidence, then any Bayesian updating rule must violate either **COMMUTATIVITY** or **HOLISM**.

- *My Aim.* I will explain **WEISBERG'S CLAIM**.

1. Holism and Commutativity

- *The Argument from Defeasibility.* Suppose we become rationally confident in a proposition on the basis of some evidence. It seems that our confidence in that proposition can decrease due to the rational impact of new evidence.

- *Urn.* Suppose ten balls were put into an opaque urn. I saw that nine of them were black while the remaining one was white. Then, I saw that a person kept randomly picking a ball from the urn and then putting it back. This happened a million times. Curiously, every time, I saw that a white ball comes up. Should I continue to be confident (or believe) that 90% of the balls in the urn are black?

Weisberg (2009) states **HOLISM** in terms of "experience." It's not clear to me that our experiences are our only source of evidence. See, e.g., Williamson (2000, ch. 9) on inferential knowledge being a source of evidence.

Weisberg puts this constraint in terms of commutativity of propositions, input distributions, and experiences. I think some of these may be misleading. For example, if you think that the contents of experiences are sometimes self-locating (i.e., about oneself or one's spatiotemporal location), there's no reason why commutativity should hold for contents or experiences in general. However, our constraint is compatible with **OCCASIONAL, PARTIAL COMMUTATIVITY ON EXPERIENCES**.

Examples of this sort are discussed in Williamson's (2000) *Knowledge and its Limits*, ch. 9.

- *The Wall*. Suppose I enter a room, and see that the wall is red. So, I justifiably believe it. When I come out of the room, my friend (whom I trust) tells me, “The wall was lit up with trick red lighting that would make any surface look red.” Should I continue to be confident (or believe) that the wall I saw was red?

In each case, the answer seems to be “No.” This suggests that how confident we are in the relevant proposition should depend on our background beliefs, such that, if the background beliefs were to rationally change, so should our confidence in that proposition. This supports HOLISM.

- *The Argument from Bias*.
 - If COMMUTATIVITY OF EVIDENCE were false, then it would be possible for an agent to intentionally and rationally manipulate her beliefs in favour of or against a hypothesis (without any loss of information or irrationality or change in epistemic standards).
 - It is impossible for an agent to intentionally *and* rationally manipulate her beliefs in favour of or against a hypothesis (without any loss of information or irrationality or change in epistemic standards).

This line of argument is found in Kelly (2008) while discussing Kripkean dogmatism.

So, COMMUTATIVITY OF EVIDENCE is true.

2. The Problem about Holism in Traditional Epistemology

Before we look at Weisberg’s dilemma, it might be worth noting that an analogue of that dilemma arises in traditional epistemology.

- *Holism for Full Belief*. For any piece of evidence E and any proposition P , there is a ‘defeater’ proposition D such that an agent’s propositional justification for believing in P upon receiving evidence E should depend on her justification in believing in D .
- *Two Theories of Justification*.
 - RELIABILISM. An agent is (propositionally) justified in believing that p iff there is a reliable method on the basis of which they could believe that p .
 - EVIDENTIALISM. An agent is (propositionally) justified in believing that p iff their total evidence supports the proposition that p to a sufficiently high degree.

This point has been discussed by many, including Lasonen-Aarnio (2010), Beddor (2015), Baker-Hytech and Benton (2015), and Srinivasan (2020).

Both theories fail to explain our intuition about *The Wall*.

- *The Problem for Reliabilism.*
 - In *The Wall*, I initially form my belief on the basis of my veridical perceptual experience. Call the method by which I hold the belief before I speak to my friend M_1 .
 - Then, I receive false testimony from my friend, and form the belief that the wall was lit up with trick red lighting. Call the relevant method M_2 .

Why should the availability of M_1 affect the reliability of M_2 ?

- *The Problem for Evidentialism.* Suppose the evidentialist accepts two claims:

THE PROPOSITIONALIST CONCEPTION OF EVIDENCE. An agent's total evidence is a proposition, or a set of propositions.

THE ENTAILMENT-SUPPORT PRINCIPLE. If an agent's total evidence entails a proposition that p , then their total evidence supports that p to a maximal degree.

Suppose that, in *The Wall*, when I learn that the wall is red, my evidence comes to entail that it is red. Then, the evidentialist has two options.

- Evidentialists could accept both the PROPOSITIONALIST CONCEPTION OF EVIDENCE or the ENTAILMENT-SUPPORT PRINCIPLE. But they can only explain how I lose justification in this case by giving us a theory of rational evidence loss. It's not obvious what that theory will look like.
- Or, they could reject either THE PROPOSITIONALIST CONCEPTION OF EVIDENCE, or THE ENTAILMENT-SUPPORT PRINCIPLE. Once again, it's not clear what we should replace these principles with.

3. Two Bayesian Updating Rules

- *A Presupposition of Standard Bayesianism.* Orthodox Bayesians are committed to:

STRICT CONDITIONALIZATION. Let t_1 and t_2 be two times such that, between those two times, the strongest evidence an agent receives is E . Then, if the agent's credence function is $Pr_1(\cdot)$ and $Pr_1(E) > 0$, then her credence function $Pr_2(\cdot)$ at t_2 should just be the conditional probability function $Pr_1(\cdot|E)$.

Since E here is a proposition, this commits us to THE PROPOSITIONALIST CONCEPTION OF EVIDENCE.

- *The Argument for Propositionalism.*

More clearly, either I can retain my belief about the colour of the wall on the basis of M_1 , or I can't. If I can, then my belief will be justified. It's unclear why the availability of M_2 should affect the reliability of M_2 . If I can't, the reliabilist owes us an explanation of why the method by which my belief is held should change from M_1 to something else. For this line of reasoning, see Lasonen-Aarnio (2010).

The problems here roughly take the same form as the problems for AGM.

Williamson offers an argument like this in KAIL, ch. 10.

- Relations of evidential support hold between an agent's evidence and certain hypotheses.
- Relations of evidential support are explanatory, probabilistic and logical relations that can only hold between propositions.
- Therefore, an agent's evidence must either be a proposition or a set of propositions.
- *Jeffrey's Argument Against Propositionalism.*
 - If the PROPOSITIONALIST CONCEPTION OF EVIDENCE is true, then an agent should update her credences by becoming certain about her evidence and then adjusting the rest of her credences (as STRICT CONDITIONALIZATION requires).
 - If that is right, then the credences of a rational agent must always be based on her certainty regarding a number of propositions.
 - A rational agent's credences needn't be based on her certainty regarding any proposition.
 - Therefore, PROPOSITIONALISM is false.
- *Jeffrey's Example.*

An agent is examining a cloth by dim candlelight. On the basis of this experience, the agent sharply (and rationally) raises her credence in the cloth's being green. But she also acknowledges that the cloth could be blue, or perhaps even violet. So, the agent does not assign probability 1 to the proposition G that the cloth is green.

Two observations:

- If this instance of experiential learning is to fit STRICT CONDITIONALIZATION, the agent's new credence in G must be the result of indirect learning by conditionalization on some other proposition E to which the agent did assign probability 1.
- What would that other proposition be? The obvious candidates, such as the proposition that the cloth looks green or possibly blue or conceivably violet," cover a wide range of different experiences that would justify a wide range of different probabilities for G .

The upshot: there seems to be no proposition that the agent can conceptually distinguish, such that it is made certain by the experience, and is precise enough to capture the evidential import of the experience.

- *Jeffrey's solution.*

The argument can be found in Jeffrey ((1965), p. 165).

This seems like a bad argument to me. See Williamson (?, ch. 9) for more discussion.

- When we undergo an experience, we don't receive any evidence in the form of a proposition or a set of propositions. Rather, as a result of the experience, certain constraints are imposed on our posterior credence function.
- In response to these constraints, we must rationally adjust our other credences. So, these constraints on our posterior credences may be thought of as our evidence.

Following Jeffrey, we may represent these constraints as a probability distribution over a partition.

THE PROBABILISTIC CONCEPTION OF EVIDENCE. Whenever an agent gains new evidence, her evidence can be represented as an input probability distribution C over a partition Π over a set of possible worlds W .

For example, in the candlelight situation, we may imagine that, when the agent looks at the cloth, the new evidence that she gains can be represented as a probability distribution C over a partition containing $\{G, \sim G\}$ such that $C(G) = 0.9$.

- *Jeffrey's Rule.* In accordance with this probabilistic conception of evidence, Jeffrey proposes a new rule of updating.

JEFFREY CONDITIONALIZATION. Suppose, between t_1 and t_2 , the evidence that an agent receives is a probability distribution C over a partition $\Pi = \{E_1, \dots, E_k\}$. Then, if Pr_1 is their credence function at t_1 , then their posterior credences in any proposition X should be:

$$Pr_2(X) = \sum_i Pr_1(X|E_i)C(E_i).$$

- *An Example.* Consider a simple version of the candlelight example.
 - Suppose the evidence the agent receives when they look at the cloth can be represented as a probability distribution C over the partition $\{G, \sim G\}$ such that $C(G) = 0.9$ and $C(\sim G) = 0.1$.
 - Consider the proposition *Silk*, i.e., the proposition that the cloth is made of silk. Let's say that, by lights of the agent's prior credences, it's quite likely that the cloth is made of silk, given that it's green. So, $Pr_1(\text{Silk}|G) = 0.8$. But it's quite unlikely that it's made of silk, given that it's not green. So, $Pr_1(\text{Silk}|\sim G) = 0.2$.
 - What should the agent's posterior credence in *Silk* be?

$$\begin{aligned} Pr_2(\text{Silk}) &= Pr_1(\text{Silk}|G)C(G) + Pr_1(\text{Silk}|\sim G)C(\sim G) \\ &= 0.8 \times 0.9 + 0.2 \times 0.1 \\ &= 0.72 + 0.02 \\ &= 0.74. \end{aligned}$$

- *Relationship with Conditionalization.* STRICT CONDITIONALIZATION is an instance of JEFFREY CONDITIONALIZATION.
 - To see this, just imagine a case where $C(G) = 1$.
 - In that case, the agent’s posterior credence in *Silk* will just be her prior conditional credence in *Silk* on G .

$$\begin{aligned} Pr_2(Silk) &= Pr_1(Silk|G)C(G) + Pr_1(Silk|\sim G)C(\sim G) \\ &= Pr_1(Silk|G) \times 1 + Pr_1(Silk|\sim G) \times 0 \\ &= Pr_1(Silk|G) \\ &= 0.8. \end{aligned}$$

4. Weisberg’s Claim

4.1 Updating by Conditionalization

Suppose perception can be the source of propositional evidence. So, if BAYESIAN CONDITIONALIZATION is true, then a rational agent can become certain about various (non-trivial) propositions about the external world.

- *Commutativity.* We know that STRICT CONDITIONALIZATION clearly preserves COMMUTATIVITY OF EVIDENCE (at least if the evidence is always a true proposition).
- *Holism.* However, it is incompatible with HOLISM.
 - In cases like *Urn* and *The Wall*, if the agent gains conclusive evidence in favour of a proposition, then no change in her background beliefs can lower her credence in that proposition.
 - In other words, once she learns this proposition, her certainty in that proposition becomes completely independent of her background beliefs.

Here’s why.

- Let Pr be the agent’s prior credence function such that $Pr(E_1 \cap E_2) \neq 0$. Let this agent be a conditionalizer. Now, suppose she learns E_1 first and then learns E_2 . So, let Pr_{E_1} be the credence function she has after learning E_1 . So, for any proposition H , $Pr_{E_1}(H) = Pr(H|E_1)$.
- Now, suppose she learns E_2 . So, let her credence function be Pr_{E_1,E_2} . We know:

$$\begin{aligned} Pr_{E_1,E_2}(H) &= Pr_{E_1}(H|E_2) \\ &= \frac{Pr_{E_1}(H \cap E_2)}{Pr_{E_1}(E_2)} \\ &= \frac{Pr(H \cap E_2|E_1)}{Pr(E_2|E_1)} \\ &= \frac{Pr((H \cap E_2) \cap E_1)}{Pr(E_2 \cap E_1)} \\ &= Pr(H|E_1 \cap E_2) \end{aligned}$$

- Now, if we reverse the order of learning, then we can let Pr_{E_2} be the credence function she has after learning E_2 . If she now learns E_1 , her credence function will be Pr_{E_2,E_1} . By similar reasoning as above, $Pr_{E_2,E_1}(H) = Pr(H|E_1 \cap E_2)$.

4.2 Updating by Jeffrey Conditionalization

Suppose we accept the PROBABILISTIC CONCEPTION OF EVIDENCE, and say that perceptual experiences can only ever make it rational for us to adopt non-extremal credences in contingent hypotheses.

- *Holism.* If that is right, then JEFFREY CONDITIONALIZATION is indeed compatible with HOLISM.
 - In cases like *Urn* and *The Wall*, suppose I gain evidence that makes me highly confident that the wall is red or that nine of the balls are black while the remaining one is white.
 - But, then, there indeed are other propositions such that, if I were to become highly confident in them, my confidence in these propositions would be lowered.
- *An Example.* To see this, suppose that my original experience provides me with evidence that can be represented with the input distribution C_1 over $\{H, \sim H\}$, where

- H is the proposition that there are nine black balls and one white ball in the urn.
- $C_1(H) = 0.99$.

Let my credence function at this stage be Pr_1 . Next, I undergo a series of experiences that collectively provide me with evidence that can be represented with the input distribution C_2 over $\{E, \sim E\}$, where

- E is the proposition that a white ball is drawn randomly a million times.
- $C_2(E) = 0.99$.

Let my credence function at this stage be Pr_2 . In that case, my final credence in H should be:

$$Pr_2(H) = Pr_1(H|E)C_2(E) + Pr_1(H|\sim E)C_2(\sim E)$$

Since both $Pr_1(H|E)$ and $C_2(\sim E)$ are quite low, $Pr_2(H)$ should be quite low.

- *Commutativity.* The problem is that, if we accept the **PROBABILISTIC CONCEPTION OF EVIDENCE**, we will violate **COMMUTATIVITY OF EVIDENCE**. For, if we reverse the order in which I get the two bits of evidence in the example above, then I will end up being confident that the urn contains nine black balls and one white ball.
- *A Way Out?* As Field (1978) and Wagner (2002) noticed, we can preserve **COMMUTATIVITY OF EVIDENCE** by adopting the following conception of evidence.

See, also, Lange (2000).

THE REVISED PROBABILISTIC CONCEPTION OF EVIDENCE. Suppose an agent with a prior probability function Pr_1 gains new evidence, such that it is rational for her to adopt a credence function Pr_2 in response to that evidence. Then, her evidence can be represented as Bayes factors defined over a partition Π , such that for any two cells $E_i, E_j \in \Pi$, the Bayes factor corresponding to them is:

$$B(E_i : E_j) = \frac{Pr_2(E_i)/Pr_2(E_j)}{Pr_1(E_i)/Pr_1(E_j)}.$$

The Problem. This view predicts that, if an agent were to receive the same evidence over and over again, she can keep in principle rationally increasing or decreasing her credences in the relevant proposition. That's bad.

4.3 A Diagnosis

- *Rigidity.* **STRICT CONDITIONALIZATION** and **JEFFREY CONDITIONALIZATION** are both rigid, i.e., they preserve the conditional

probabilities on the propositions that the evidence directly bears on.

- When we apply STRICT CONDITIONALIZATION to evidence E , $Pr_1(H|E) = Pr_2(H|E)$.
- Similarly, if we apply Jeffrey Conditionalization to the partition Π , then, for any $E \in \Pi$, $Pr_1(H|E) = Pr_2(H|E)$.
- *An Interesting Result.* Suppose that an updating rule that takes a probability function Pr_1 and outputs Pr_2 is rigid with respect to the partition $\{E, \sim E\}$. Then the following two conditions are incompatible:
 - $Pr_1(E|F) = Pr_1(E)$.
 - $Pr_2(E|F) < Pr_2(E)$.

In particular, $Pr_1(E|F) = Pr_1(E)$ entails $Pr_2(E|F) = Pr_2(E)$.

- *Why Is This Significant?* It shows that, if a rule is rigid, then undercutting defeat is impossible for some propositions.
 - Suppose an agent undergoes an experience, and thereby becomes rationally confident in E .
 - Then, she undergoes another experience and then becomes rationally confident in F . If this newly gained confidence in F is to rationally undermine her confidence in E without directly telling against it, it has to be the case that $Pr_2(E|F) < Pr_2(E)$.

But, if it was originally the case that $Pr_1(E|F) = Pr_1(E)$, then this cannot happen.

5. Escape Strategies

- *A Problem.* Notice that WEISBERG'S CLAIM creates a problem for Bayesians.
 - CLAIM 1. HOLISM and COMMUTATIVITY are true.
 - CLAIM 2. If perceptual experiences can be the source of evidence, then Bayesian theories of belief revision cannot accommodate HOLISM and COMMUTATIVITY.

So, either perceptual experiences cannot be the source of evidence, or Bayesians are wrong about rational belief revision.

- *Responses.*
 - *Strategy 1.* Perceptual experiences aren't a source of evidence.

- *Strategy 2*. There are other Bayesian rules of belief revision that are both commutative and holistic (Gallow (2014)).
- *Strategy 3*. If we combine Bayesianism with a theory of evidence loss (or a form of contextualism), we can accommodate HOLISM and COMMUTATIVITY (Greco Greco2017).

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