The Plurality Inference as a Quantity Implicature

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1 Plurality Inference and Unmarked Plural

Plural noun phrases typically have **plurality inferences** (alt.: **multiplicity inferences**).

(1) I saw *foxes* in the garden.

In negative contexts, they behave as if they are number neutral (**unmarked plural**) (Sauerland 2003, Sauerland, Anderssen & Yatsushiro 2005, among others):

(2) a. I did<u>n't</u> see *foxes* in the garden.
b. <u>If</u> I see foxes in the garden, I'll let you know.
c. I left <u>before</u> *philosophers* arrived.

With certain quantificational contexts they give rise to **partial plurality inferences**.

(3) a. Exactly one of us saw *foxes* in the garden.
b. Every applicant submitted *their experimental papers*.

Theories of the plurality inference postulate number-neutral meaning for plural noun phrases and derive the plurality inference by some other means.

- Scalar implicature approach (Spector 2007, Zweig 2009, Ivlieva 2013, Mayr 2015)
  The plurality inference is a scalar implicature.
- Ambiguity approach (Farkas & de Swart 2010, Grimm 2013, Martí 2018)
  Ambiguity between plural and number-neutral meaning. But this cannot deal with (3).
- Homogeneity theory (Križ 2017)
  (2) and (3) are due to homogeneity.
- Anti-presupposition approach (Sauerland 2003, Sauerland et al. 2005)
  The plurality inference is an anti-presupposition.

2 A New Scalar Implicature Approach

Assumptions from previous analyses:

- The plural is semantically number neutral. This straightforwardly accounts for the (2).
- The plurality inference arises as a scalar implicature in competition with the singular.

But the literal meanings of (4a) and (4b) will be truth-conditionally equivalent on the assumption that the plural is semantically number neutral.

(4) a. I saw foxes. b. I saw a fox.

In order to generate a scalar implicature, there needs to be some semantic asymmetry between these two sentences.
Two proposals from the literature:

- Higher-order scalar implicatures (Spector 2007):
  The plural competes with the singular that has its own scalar implicature.

- Local scalar implicatures (Zweig 2009, Ivlieva 2013, Mayr 2015):
  The two sentences are truth-conditionally distinct at some sub-constituents.

I propose a new scalar implicature account that doesn't require these additional mechanisms (though I do not necessarily deny them).

1. Scalar implicatures can arise from non-propositional aspects of meaning.

   The Gricean Maxim of Quantity is about informativity in general, and should apply to non-propositional aspects of meaning as well.

   (5) a. Make your contribution as informative as is required (for the current purposes of the exchange).
       b. Do not make your contribution more informative than is required.

2. Indefinites introduce **discourse referents** (Karttunen 1976):

   (6) a. Bill saw **a unicorn**. The unicorn had a gold mane.
       b. Bill didn't see **a unicorn**. *The unicorn had a gold mane.

   Let us say that the appearance of an indefinite noun phrase establishes a "discourse referent" just in case it justifies the occurrence of a coreferential pronoun or a definite noun phrase later in the text. (Karttunen 1976:366)

   Information about discourse referents is part of the meaning of a sentence, distinct from its propositional/truth-conditional content.

   (7) a. John has a wife. She is French.
       b. ??John is married. She is French.

   (8) a. One of the ten marbles is not in the bag. It's probably under the sofa.
       b. Nine of the ten marbles are in the bag. ??It's probably under the sofa.

   Karttunen (1976) discusses what kind of operators 'kill' discourse referents.

   - This idea was later formalized in dynamic semantics (Kamp 1981, Heim 1983).
   - Modern versions of dynamic semantics talks about discourse referents introduced by various quantifiers (van den Berg 1996, Nouwen 2003a, Brasoveanu 2007):

     (9) a. Exactly one^x student passed. She$_x$ solved a difficult problem.
         b. Most^x students passed. They$_x$ studied hard.

     as well as complex interactions between indefinites and quantifiers ('quantificational subordination'):

     (10) Every PhD student of mine wrote a^x long paper.
          a. *It$_x$ is about anaphora.
          b. They all submitted it$_x$ to *Journal of Semantics*.

2.1 The Plurality Inference as a Quantity Implicature

Claim: the plurality inference is a quantity implicature about possible values of discourse referents.

E.g. (4a) and (4b) have the same truth-conditions but differ in what kind of discourse referents they introduce.

(4′) a. I saw foxes^x. b. I saw a^x fox.

- (4′a) introduces a discourse referent x ranging over both singular and plural entities.
- (4′b) introduces a discourse referent x ranging only over singular entities.

(4′b) yields fewer possible values of x than (4′a), so it's stronger/more informative.
I make use of this semantic asymmetry to generate a (secondary) scalar implicature that what (4′b) would have meant is not what the speaker intended to mean. Consequently, the discourse referent should range only over plural entities.[1]

2.2 Negation

This analysis straightforwardly accounts for the behaviour under negation:

(11) a. I didn't see foxes. b. I didn't see a fox.

Neither of these sentences introduce a discourse referent at the global level, so their meanings are completely identical. As a consequence, there's no semantic asymmetry, and no scalar implicature.

2.3 Quantifiers

One empirical advantage of the scalar implicature theories is that it accounts for partial plurality inferences in quantified sentences (Spector 2007, Ivlieva 2013).

(12) a. Exactly one of us saw saw foxes. b. Exactly one of us saw a fox.

The present account can deal with this.

- (12a) introduces two discourse referents, one ranging over individuals among us, and one ranging over singular or plural foxes.
- (12b) introduces two discourse referents, one ranging over individuals among us, and one ranging over singular foxes.

The (secondary) implicature amounts to that the second discourse referent does not range over singular papers.

3 Update Semantics

Sentences are translated into formulas, which are interpreted as functions from contexts to contexts ('Context Change Potentials').

3.1 Basic Update Semantics

Definition 1. (Contexts and Assignments)

- A *context* is a set of pairs consisting of a possible world w and an assignment a, representing the shared beliefs among the discourse participants.
- An *assignment* is a total function from variables to the domain D of the model $\mathcal{M}$.[2]
- The possible worlds of context c, $\mathbf{W}_c := \{\, w \mid \text{for some } \langle w, a\rangle \in c \,\}$.

[1] Van Rooij (2017) pursues related ideas in a similar framework, but he crucially only derives scalar implicature based on truth-relevant concepts, i.e. possible worlds/truth-makers, and his analysis suffers from problems with simple cases of scalar implicatures like *some*.

[2] Nothing crucial hinges on this. We could use partial functions instead of total functions.

- The assignments of context c, $\mathbf{A}_c \coloneqq \{\, a \mid \text{for some } \langle w, a\rangle \in c \,\}$.

Definition 2. (Update Rules) For any model $\mathcal{M} = \langle D, W, I\rangle$:

$$[\mathtt{t}]^a_{\mathcal{M}} \coloneqq \begin{cases} I(\mathtt{t}) & \text{if } \mathtt{t} \text{ is a constant} \\ a(\mathtt{t}) & \text{if } \mathtt{t} \text{ is a variable} \end{cases}$$

$$c[\mathrm{P}(\mathtt{t}_1, \dots, \mathtt{t}_n)]_{\mathcal{M}} \coloneqq \{\, \langle w, a\rangle \in c \mid \langle [\mathtt{t}_1]^a, \dots, [\mathtt{t}_n]^a)\rangle \in I(w, \mathrm{P}) \,\}$$

$$c[\neg\varphi]_{\mathcal{M}} \coloneqq \{\, \langle w, a\rangle \in c \mid \{\, \langle w, a\rangle \,\}\, [\varphi] = \varnothing \,\}$$

$$c[(\varphi \wedge \psi)]_{\mathcal{M}} \coloneqq c[\varphi][\psi]$$

$$c[(\mathtt{t}_1 = \mathtt{t}_2)]_{\mathcal{M}} \coloneqq \{\, \langle w, a\rangle \in c \mid [\mathtt{t}_1]^a = [\mathtt{t}_2]^a \,\}$$

(13) Andrew sat down. $\rightsquigarrow \mathtt{SatDown}(\mathtt{andrew})$

3.2 Indefinites and Random Assignment

Indefinites trigger *random assignment* ($\exists\mathtt{x}$ is taken to be a formula), which resets the possible values of $\mathtt{x}$.

Definition 3. (Random Assignment) We'll write '$a[\mathtt{x} \mapsto e]$' to mean that assignment that differs from a at most in that it maps variable $\mathtt{x}$ to entity e.

$$c[\exists\mathtt{x}] \coloneqq \{\, \langle w, a[\mathtt{x} \mapsto e]\rangle \mid e \in D \text{ and } \langle w, a\rangle \in c \,\}$$

This semantics accounts for cross-sentential anaphora with an indefinite antecedent:

(14) $\text{A}^{\mathtt{x}}$ farmer walked in. $\text{He}_{\mathtt{x}}$ sat down.
$\rightsquigarrow (\exists\mathtt{x} \wedge \mathtt{Farmer}(\mathtt{x}) \wedge \mathtt{WalkedIn}(\mathtt{x})) \wedge \mathtt{SatDown}(\mathtt{x})$

$\exists\mathtt{x}$ randomly introduces new values for $\mathtt{x}$, and $\mathtt{Farmer}(\mathtt{x}) \wedge \mathtt{WalkedIn}(\mathtt{x})$ discards those world-assignment pairs that do not assign $\mathtt{x}$ a farmer who walked in in the respective possible worlds. Then, $\mathtt{SatDown}(\mathtt{x})$ will operate on the resulting set of world-assignment pairs, and eliminate those pairs that assign $\mathtt{x}$ an entity that did not sit down in the respective possible worlds.

3.3 Plural Entities

We will allow variables to range over plural entities in addition to singular entities.[3]

We assume that from any two entities e and f in the domain of the model, a new entity $e \oplus f$ can be formed that has e and f (and nothing else) as parts, and all of these entities (and only they) are members of the domain of the model (Link 1983).[4]

Predicates are also specified for plurality in the standard way. Crucially, we assume that plural nouns are semantically unmarked and number neutral. A predicate like $\mathtt{Farmers}$ is inherently distributive:

(15) a. $e \in I(w, \mathtt{Farmer}) \Leftrightarrow e$ is a singular entity and e is a farmer in w
b. $e \in I(w, \mathtt{Farmers}) \Leftrightarrow$ each singular part of e is a farmer in w

(We won't talk about non-distributive predicates, but the semantics is compatible with them.)

Recall that under the assumption that the plural is semantically unmarked, the following two sentences have the same truth-conditions.

[3] See Van den Berg (1996:Ch.3) for a similar idea. He eventually proposes a different way to deal with pluralities where they are encoded in a set of assignments. We'll come back to this in §5.

[4] Nothing crucial hinges on the use of individual sums. Sets could be used instead (Landman 1989a,b, Schwarzschild 1996, Winter 2000).

(16) a. Andrew wrote a$^{\mathtt{x}}$ paper. $\rightsquigarrow \exists \mathtt{x} \wedge \mathtt{Wrote}(\mathtt{andrew}, \mathtt{x}) \wedge \mathtt{Paper}(\mathtt{x})$
b. Andrew wrote papers$^{\mathtt{x}}$. $\rightsquigarrow \exists \mathtt{x} \wedge \mathtt{Wrote}(\mathtt{andrew}, \mathtt{x}) \wedge \mathtt{Papers}(\mathtt{x})$

However, their dynamic denotations are different. We'll write $a \approx_{\mathtt{x}} b$ to mean that the two assignments a and b differ at most in the value for $\mathtt{x}$.

(17) a. $$c[\exists \mathtt{x} \wedge \mathtt{Wrote}(\mathtt{andrew}, \mathtt{x}) \wedge \mathtt{Paper}(\mathtt{x})] = \left\{ \langle w, b \rangle \;\middle|\; \begin{array}{l} \text{for some } \langle w, a \rangle \in c,\ a \approx_{\mathtt{x}} b \text{ and} \\ \text{Andrew wrote } b(\mathtt{x}) \text{ in } w \text{ and} \\ b(\mathtt{x}) \text{ is singular and } b(\mathtt{x}) \text{ is a paper in } w \end{array} \right\}$$

b. $$c[\exists \mathtt{x} \wedge \mathtt{Wrote}(\mathtt{andrew}, \mathtt{x}) \wedge \mathtt{Papers}(\mathtt{x})] = \left\{ \langle w, b \rangle \;\middle|\; \begin{array}{l} \text{for some } \langle w, a \rangle \in c,\ a \approx_{\mathtt{x}} b \text{ and} \\ \text{Andrew wrote each singular part of } b(\mathtt{x}) \text{ in } w \text{ and} \\ \text{each singular part of } b(\mathtt{x}) \text{ is a paper in } w \end{array} \right\}$$

In (17b), $b(\mathtt{x})$ can be a plural entity, but not in (17a).[5] We will make use of this semantic asymmetry to derive the plurality inference.

(18) Assume the following worlds:
- w_1: Andrew wrote p_1 and no other papers.
- w_2: Andrew wrote p_1 and p_2 and no other papers.
- w_3: Andrew wrote no papers.

and $c = \{ \langle w_1, a \rangle, \langle w_1, b \rangle, \langle w_2, a \rangle, \langle w_2, d \rangle, \langle w_3, d \rangle \}$.

$$c[\exists \mathtt{x} \wedge \mathtt{Wrote}(\mathtt{andrew}, \mathtt{x}) \wedge \mathtt{Paper}(\mathtt{x})] = \left\{ \begin{array}{l} \langle w_1, a[\mathtt{x} \mapsto p_1] \rangle, \langle w_1, b[\mathtt{x} \mapsto p_1] \rangle, \\ \langle w_2, a[\mathtt{x} \mapsto p_1] \rangle, \langle w_2, d[\mathtt{x} \mapsto p_1] \rangle, \\ \langle w_2, a[\mathtt{x} \mapsto p_2] \rangle, \langle w_2, d[\mathtt{x} \mapsto p_2] \rangle \end{array} \right\}$$

$$c[\exists \mathtt{x} \wedge \mathtt{Wrote}(\mathtt{andrew}, \mathtt{x}) \wedge \mathtt{Papers}(\mathtt{x})] = \left\{ \begin{array}{l} \langle w_1, a[\mathtt{x} \mapsto p_1] \rangle, \langle w_1, b[\mathtt{x} \mapsto p_1] \rangle, \\ \langle w_2, a[\mathtt{x} \mapsto p_1] \rangle, \langle w_2, d[\mathtt{x} \mapsto p_1] \rangle, \\ \langle w_2, a[\mathtt{x} \mapsto p_2] \rangle, \langle w_2, d[\mathtt{x} \mapsto p_2] \rangle, \\ \langle w_2, a[\mathtt{x} \mapsto p_1 \oplus p_2] \rangle, \langle w_2, d[\mathtt{x} \mapsto p_1 \oplus p_2] \rangle \end{array} \right\}$$

4 Scalar Implicature in Update Semantics

4.1 How to Derive the Plurality Inference

Let $c_{\mathsf{sg}} = c[\exists \mathtt{x} \wedge \mathtt{Wrote}(\mathtt{andrew}, \mathtt{x}) \wedge \mathtt{Paper}(\mathtt{x})]$ and $c_{\mathsf{pl}} = c[\exists \mathtt{x} \wedge \mathtt{Wrote}(\mathtt{andrew}, \mathtt{x}) \wedge \mathtt{Papers}(\mathtt{x})]$. Whenever c_{sg} and c_{pl} are non-empty and non-equivalent (that is, there are worlds in $\mathbf{W}_c$ in which Andrew wrote more than one paper), there is an asymmetric relation, namely:

$$c_{\mathsf{sg}} \subset c_{\mathsf{pl}}$$

Note that $\mathbf{W}_{c_{\mathsf{sg}}} = \mathbf{W}_{c_{\mathsf{pl}}}$, because the two sentences are truth-conditionally equivalent. The crucial difference is coming from the assignment functions, i.e. $\mathbf{A}_{c_{\mathsf{sg}}} \subset \mathbf{A}_{c_{\mathsf{pl}}}$.
I propose that the plurality inference is derived by subtracting c_{sg} from c_{pl}. For example, in the case of (17), we get as a result:

$$\{ \ \langle w_2, a[\mathtt{x} \mapsto p_1 \oplus p_2] \rangle, \langle w_2, d[\mathtt{x} \mapsto p_1 \oplus p_2] \rangle \ \}$$

[5]Note that according this semantics, *Andrew wrote a paper* is compatible with him having written more than one paper (as in w_2 above). This is fine as *a*-indefinites are generally non-maximal. Its maximal reading, if it's available, needs to be derived by some other means, possibly as an implicature of some kind (see Spector 2007). We will not deal with this here.

In w_2, Andrew wrote two papers and all the values for $\mathtt{x}$ are pluralities consisting of multiple papers.

More generally, let c' be the resulting context after the above subtraction operation. Then, whenever $c' \neq \emptyset$, Andrew wrote multiple papers in $\mathbf{W}_{c'}$ and for each $a \in \mathbf{A}_{c'}$, $a(\mathtt{x})$ is a plural entity. This is the plurality inference.

Consequently, we account for the fact that the plural papers can be referred back to later in the discourse by a plural pronoun.

(19) Andrew wrote papers$^{\mathtt{x}}$. They$_{\mathtt{x}}$ are about Slovenian duals.

4.2 Scalar Implicature and Informativity in Dynamic Semantics

The Gricean Maxim of Quantity says:

(20) a. Make your contribution as informative as is required (for the current purposes of the exchange).
b. Do not make your contribution more informative than is required.

The notation of 'informativity' is often understood in terms of truth-conditional entailment:

(21) φ is truth-conditionally more informative than ψ iff φ entails ψ but ψ does not entail φ (alt.: φ asymmetrically entails ψ).

In our update semantics, this can be paraphrased as follows:

Definition 4. (Truth-Conditional Informativity) φ is *truth-conditionally more informative* than ψ iff for each context c, $\mathbf{W}_{c[\varphi]} \subseteq \mathbf{W}_{c[\psi]}$ but in some context c', $\mathbf{W}_{c'[\psi]} \nsubseteq \mathbf{W}_{c'[\varphi]}$.

In update semantics we can have a different notation of informativity:

Definition 5. (Dynamic Informativity) φ is *dynamically more informative* than ψ iff for each context c, $c[\varphi] \subseteq c[\psi]$ but in some context c', $c'[\psi] \nsubseteq c'[\varphi]$.

This is distinct from truth-conditional informativity, because the asymmetry may come from the anaphoric potentials encoded in the assignments. And this is exactly what we use to derive the plurality inference.

I formulate the scalar implicature computation as follows.[6]

(22) If φ has an alternative ψ that is dynamically more informative, then an assertion of φ in c by a cooperative speaker is interpreted as $c[\varphi] - c[\psi]$.

As we have already seen, (22) gives rise to the plurality inference, e.g. (18).

4.2.1 Negation (and other connectives)

Recall that in negative sentences plurality inferences are not observed. This is explained as follows: under negation, the singular and plural sentences have the exact same dynamic meaning. So neither of them is more informative than the other (under either notion of informativity):

(23) a. $c[\neg(\exists\mathtt{x} \wedge \mathtt{Wrote}(\mathtt{andrew}, \mathtt{x}) \wedge \mathtt{Paper}(\mathtt{x}))] = \{\, \langle w, a \rangle \in c \mid$ Andrew wrote no paper in $w \,\}$
b. $c[\neg(\exists\mathtt{x} \wedge \mathtt{Wrote}(\mathtt{andrew}, \mathtt{x}) \wedge \mathtt{Papers}(\mathtt{x}))] = \{\, \langle w, a \rangle \in c \mid$ Andrew wrote no paper in $w \,\}$

Other connectives are perhaps more complicated, and I leave them for future research for now. Some considerations:

[6] I'm only dealing with *secondary implicatures* in the sense of Sauerland (2004) here. See §6

- It's not realistic to analyze conditional as material implication (as Heim 1983 said from the beginning). Perhaps a plural in a conditional does give rise to an inference, because a conditional enables modalized crosssentential anaphora (alt.: modal subordination).

(24) If Andrew writes papers, they will be about morphology. I won't read <u>them</u>.

- Disjunction gives rise to its own scalar implicature. Here we might need embedded scalar implicature?

(25) Andrew was reading papers or writing a book.

4.2.2 *Most*

The above mechanism of scalar implicature computation works for other types of scalar implicatures more generally, e.g. *most*, which by assumption competes with *all*.

One way to deal with generalized quantifiers in dynamic semantics is via the maximality operator (Van den Berg 1996).

Definition 6. (Maximality Operator)

$$c[\mathtt{M}^{\mathtt{x}}(\varphi)] = \{\, \langle w, a\rangle \in c[\exists x \wedge \varphi] \mid \text{for no } \langle w, a'\rangle \in c[\exists x \wedge \varphi],\, a(\mathtt{x}) \sqsubset a'(\mathtt{x}) \,\}$$

In words, for each $\langle w, a\rangle \in c[\mathtt{M}^{\mathtt{x}}(\varphi)]$, a assigns a maximal value to $\mathtt{x}$ that satisfies φ in w.

This operator is useful in defining (selective) generalized quantifiers, because they can be seen as expressing relations between two maximal entities (which stand for sets in the classical setting). $\#(e)$ is the number of atomic entities in e.[7]

(26) a. $$c[\mathtt{Most}^{\mathtt{x}}(\varphi)(\psi)] = \left\{ \langle w, a\rangle \in c[\mathtt{M}^{\mathtt{x}}(\varphi \wedge \psi)] \;\middle|\; \begin{array}{l} \text{for some } \langle w, a'\rangle \in c[\mathtt{M}^{\mathtt{x}}(\varphi)] \\ \dfrac{\#(a(\mathtt{x}))}{\#(a'(\mathtt{x}))} \gg \dfrac{1}{2} \end{array} \right\}$$

b. $$c[\mathtt{All}^{\mathtt{x}}(\varphi)(\psi)] = \left\{ \langle w, a\rangle \in c[\mathtt{M}^{\mathtt{x}}(\varphi \wedge \psi)] \;\middle|\; \begin{array}{l} \text{for some } \langle w, a'\rangle \in c[\mathtt{M}^{\mathtt{x}}(\varphi)] \\ \#(a(\mathtt{x})) = \#(a'(\mathtt{x})) \end{array} \right\}$$

(27) Let us assume that in w_1, w_2, w_3 and w_4, there are exactly 10 linguists, ℓ_1, ..., ℓ_{10}.

- w_1: All linguists smoke.
- w_2: Only ℓ_1, ..., ℓ_8 smoke.
- w_3 Only ℓ_1, ..., ℓ_3 smoke.
- w_4: No linguists smoke.

a. $$\{\langle w_1, a_1\rangle, \langle w_2, a_2\rangle, \langle w_3, a_3\rangle, \langle w_4, a_4\rangle\}\,[\mathtt{Most}^{\mathtt{x}}(\mathtt{Linguists}(\mathtt{x}))(\mathtt{Smoke}(\mathtt{x}))]$$
$$= \{\langle w_1, a_1[\mathtt{x} \mapsto \ell_1 \oplus \cdots \oplus \ell_{10}]\rangle, \langle w_2, a_2[\mathtt{x} \mapsto \ell_1 \oplus \cdots \oplus \ell_8]\rangle\}$$

b. $$\{\langle w_1, a_1\rangle, \langle w_2, a_2\rangle, \langle w_3, a_3\rangle, \langle w_4, a_4\rangle\}\,[\mathtt{All}^{\mathtt{x}}(\mathtt{Linguists}(\mathtt{x}))(\mathtt{Smoke}(\mathtt{x}))]$$
$$= \{\langle w_1, a_1[\mathtt{x} \mapsto \ell_1 \oplus \cdots \oplus \ell_{10}]\rangle\}$$

The *all*-alternative is dynamically more informative. Consequently, we derive the scalar implicature, and end up with the following singleton set, as desired.

$$\{\langle w_2, a_2[\mathtt{x} \mapsto \ell_1 \oplus \cdots \oplus \ell_8]\rangle\}$$

This captures so-called *refset anaphora* (Van den Berg 1996, Nouwen 2003b). Refset anaphora is anaphora to the entities that satisfy both the restrictor and nuclear scope as in (28).

(28) Most$^{\mathtt{x}}$ linguists smoke. They$_{\mathtt{x}}$ (= the linguists who smoke) also drink.

[7] In order to deal with mass nouns, which *most* and *all* are compatible with, we need a more general definition of #.

5 Plurality Inferences in Quantificational Contexts

One of the advantages of scalar implicature theories of plurality inferences is that they account for plurality inferences in non-monotonic contexts:

(29) a. Exactly one$^{\mathrm{x}}$ linguist wrote papers$^{\mathrm{y}}$ last year.
b. Exactly one$^{\mathrm{x}}$ linguist wrote a$^{\mathrm{y}}$ paper last year.

(29a) implies that the only linguist who wrote papers last year wrote more than one paper.

Our analysis derives this inference without resorting to local computation of scalar implicatures or higher-order scalar implicatures (unlike other scalar implicature theories, e.g. Spector 2007, Ivlieva 2013).

- (29a) introduces two discourse referents, one ranging over singular linguists, and one ranging over singular or plural papers.
- (29b) introduces two discourse referents, one ranging over singular linguists, and one ranging over singular papers.

The latter is dynamically stronger.

5.1 Plural Information States

In order to deal with (29), we need to be able to encode the dependency between two variables, $\mathtt{x}$ and $\mathtt{y}$. We will adopt Van den Berg's (1996) *plural information states* (see also Nouwen 2003b, 2007, Brasoveanu 2007, 2008, 2010).

The idea is to model contexts as a pair consisting of a possible world and a *set* of assignments. Following Brasoveanu (2008) and Dotlačil (2013) we allow assignments to return plural entities.

Definition 7. (Assignments and contexts)

- The domain D of the model $\mathcal{M}$ is closed under sum-formation $\oplus$.
- An *assignment* is a total function from variables to D.
- A *context* is a set of pairs consisting of a possible world w and a set A of assignments.
- The possible worlds of a context $\mathbf{W}_c$ is defined as $\{\, w \mid \text{for some } \langle w, A\rangle \in c \,\}$.
- The assignment sets of a context $\mathbf{A}_c$ is defined as $\{\, A \mid \text{for some } \langle w, A\rangle \in c \,\}$.

Definition 8. (Plural Dynamic Semantics)

$$[\mathtt{t}]^{A}_{\mathcal{M}} := \begin{cases} I(\mathtt{t}) & \text{if } \mathtt{t} \text{ is a constant} \\ \bigoplus \{\, a(\mathtt{t}) \mid a \in A \,\} & \text{if } \mathtt{t} \text{ is a variable} \end{cases}$$

$$\begin{aligned} c[\mathrm{P}(\mathtt{t}_1, \dots, \mathtt{t}_n)]_{\mathcal{M}} &:= \{\, \langle w, A\rangle \in c \mid \langle [\mathtt{t}_1]^{A}, \dots, [\mathtt{t}_n]^{A})\rangle \in I_w(\mathrm{P}) \,\} \\ c[\neg\varphi]_{\mathcal{M}} &:= \{\, \langle w, A\rangle \in c \mid \{\, \langle w, A\rangle \,\}[\varphi] = \varnothing \,\} \\ c[(\varphi \wedge \psi)]_{\mathcal{M}} &:= c[\varphi][\psi] \\ c[(\mathtt{t}_1 = \mathtt{t}_2)]_{\mathcal{M}} &:= \{\, \langle w, A\rangle \in c \mid [\mathtt{t}_1]^{A} = [\mathtt{t}_2]^{A} \,\} \end{aligned}$$

From now on, we will write $A(\mathtt{x})$ instead of $\bigoplus \{\, a(\mathtt{x}) \mid a \in A \,\}$.

Definition 9. (Random Assignment)

$$c[\exists \mathtt{x}] := \left\{ \langle w, B\rangle \;\middle|\; \begin{array}{l} \text{for some } \langle w, A\rangle \in c, \\ \text{for each } a \in A, \text{ there is } b \in B \text{ such that } a \approx_{\mathtt{x}} b, \text{ and} \\ \text{for each } b \in B, \text{ there is } a \in A \text{ such that } a \approx_{\mathtt{x}} b \end{array} \right\}$$

We keep the same assumptions about the semantics of nouns as before:

(30) a. $e \in I(w, \texttt{Farmer}) \;\; \Leftrightarrow e$ is an singular entity and is a farmer in w
b. $e \in I(w, \texttt{Farmers}) \Leftrightarrow$ each singular part of e is a farmer in w

$$\begin{aligned} c[\texttt{Farmer}(\texttt{x})] &= \{ \langle w, A \rangle \in c \mid A(\texttt{x}) \in I_w(\texttt{Farmer}) \} \\ &= \left\{ \langle w, A \rangle \in c \;\middle|\; \begin{array}{l} \text{for any } a, a' \in A,\, a(\texttt{x}) = a'(\texttt{x}) \text{ and} \\ \text{the unique } e = a(\texttt{x}) \text{ for any } a \in A \text{ is a farmer in } w \end{array} \right\} \\ c[\texttt{Farmers}(\texttt{x})] &= \{ \langle w, A \rangle \in c \mid A(\texttt{x}) \in I_w(\texttt{Farmers}) \} \end{aligned}$$

Definition 10. (Maximality Operator)

$$c[\texttt{M}^{\texttt{x}}(\varphi)] := \{ \langle w, A \rangle \in c[\exists \texttt{x} \wedge \varphi] \mid \text{for no } \langle w, A' \rangle \in c[\exists \texttt{x} \wedge \varphi],\, A(\texttt{x}) \sqsubset A'(\texttt{x}) \}$$

5.2 'Exactly One'

(31) $c[\texttt{ExactlyOne}^{\texttt{x}}(\varphi)(\psi)] = \{ \langle w, A \rangle \in c[\texttt{M}^{\texttt{x}}(\varphi \wedge \psi)] \mid \#(A(\texttt{x})) = 1 \}$

(32) Exactly one$^{\text{x}}$ linguist smokes. $\rightsquigarrow \texttt{ExactlyOne}^{\texttt{x}}(\texttt{Linguist}(\texttt{x}))(\texttt{Smokes}(\texttt{x}))$

(33) Consider the following possible worlds.

- w_1: Andrew, a linguist, smokes. No other linguist smokes.
- w_2: Bill, a linguist, smokes. No other linguist smokes.
- w_3: Andrew and Bill, both linguists, smoke. No other linguist smokes.
- w_4: No linguist smokes.

Firstly, let:

$$\left\{ \begin{array}{l} \langle w_1, A_1 \rangle, \langle w_2, A_2 \rangle, \\ \langle w_3, A_3 \rangle, \langle w_4, A_4 \rangle \end{array} \right\} [\texttt{M}^{\texttt{x}}(\texttt{Linguist}(\texttt{x}) \wedge \texttt{Smokes}(\texttt{x}))] = c'$$

For each $\langle w, A \rangle \in c'$, and for each $a \in A$, a must be an atomic entity that is a linguist in w and smokes in w. So w_4 will not be in $\mathbf{W}_{c'}$.
But we will still have pairs like $\langle w_3, A \rangle$, as long as each $a \in A$ assigns $\texttt{x}$ either Andrew or Bill, and but not Andrew $\oplus$ Bill, but at the same time, it's required that $A(\texttt{x}) =$ Andrew $\oplus$ Bill, due to the maximality operator.
The number restriction of *exactly one* filters out the pairs whose world is w_3.

$$\begin{aligned} &\left\{ \begin{array}{l} \langle w_1, A_1 \rangle, \langle w_2, A_2 \rangle, \\ \langle w_3, A_3 \rangle, \langle w_4, A_4 \rangle \end{array} \right\} [\texttt{ExactlyOne}^{\texttt{x}}(\texttt{Linguist}(\texttt{x}))(\texttt{Smokes}(\texttt{x}))] \\ &= \{ \langle w, A \rangle \in c' \mid \#(A(\texttt{x})) = 1 \} \\ &= \{ \langle w_1, A_1' \rangle \in \{ w_1, A_1 \} [\exists x] \mid A_1'(\texttt{x}) = \text{Andrew} \} \cup \{ \langle w_2, A_2' \rangle \in \{ w_2, A_2 \} [\exists x] \mid A_2'(\texttt{x}) = \text{Bill} \} \end{aligned}$$

5.3 Plurality Inferences in Non-monotonic Contexts

(34) a. Exactly one$^{\text{x}}$ linguist wrote papers$^{\text{y}}$ (last year).
$\rightsquigarrow \texttt{ExactlyOne}^{\texttt{x}}(\texttt{Linguist}(\texttt{x}))(\exists \texttt{y} \wedge \texttt{Papers}(\texttt{y}) \wedge \texttt{Wrote}(\texttt{x}, \texttt{y}))$
b. Exactly one$^{\text{x}}$ linguist wrote a$^{\text{y}}$ paper (last year).
$\rightsquigarrow \texttt{ExactlyOne}^{\texttt{x}}(\texttt{Linguist}(\texttt{x}))(\exists \texttt{y} \wedge \texttt{Paper}(\texttt{y}) \wedge \texttt{Wrote}(\texttt{x}, \texttt{y}))$

The idea is the same as before. The singular version (34b) is dynamically more informative than the plural version (34a), although they are truth-conditionally equivalent. This gives rise to an implicature that $\texttt{y}$ is assigned a plural entity as its value.

(35) Consider the following worlds.

- w_1: Andrew, a linguist, wrote exactly one paper, p_1. No other linguist wrote any paper.
- w_2: Andrew, a linguist, wrote exactly two papers, p_1 and p_2. No other linguist wrote any paper.
- w_3: Bill, a linguist, wrote exactly one paper, p_3. No other linguist wrote any paper.
- w_4: Bill, a linguist, wrote exactly two papers, p_3 and p_4. No other linguist wrote any paper.
- w_5: Andrew, a linguist, wrote exactly one paper, p_1 and Bill, a linguist, wrote exactly one paper, p_3.
- w_6: No linguist wrote any paper.

$$\left\{\begin{array}{l}\langle w_1, A_1\rangle, \langle w_2, A_2\rangle,\\ \langle w_3, A_3\rangle, \langle w_4, A_4\rangle,\\ \langle w_5, A_5\rangle, \langle w_6, A_6\rangle\end{array}\right\}[\mathtt{M}^{\mathtt{x}}(\mathtt{Linguist(x)} \wedge \exists \mathtt{y} \wedge \mathtt{Paper(y)} \wedge \mathtt{Wrote(x,y)})] = c'_{\text{sg}}$$

$$\left\{\begin{array}{l}\langle w_1, A_1\rangle, \langle w_2, A_2\rangle,\\ \langle w_3, A_3\rangle, \langle w_4, A_4\rangle,\\ \langle w_5, A_5\rangle, \langle w_6, A_6\rangle\end{array}\right\}[\mathtt{M}^{\mathtt{x}}(\mathtt{Linguist(x)} \wedge \exists \mathtt{y} \wedge \mathtt{Papers(y)} \wedge \mathtt{Wrote(x,y)})] = c'_{\text{pl}}$$

Note that $\mathbf{W}_{c'_{\text{sg}}} = \mathbf{W}_{c'_{\text{pl}}} = \{ w_1, w_2, w_3, w_4, w_5 \}$.

- For each $\langle w, A\rangle \in c'_{\text{sg}}$, each $a \in A$, $a(\mathrm{y})$ is either an atomic entity that is a paper in w, and furthermore, $a(\mathrm{x})$ is the author of $a(\mathrm{y})$ in w.
- For each $\langle w, A\rangle \in c'_{\text{pl}}$, each $a \in A$, $a(\mathrm{y})$ is either an atomic entity that is a paper or a plural entity that is made up of multiple papers in w, and furthermore, $a(\mathrm{x})$ is the author of $a(\mathrm{y})$ in w.

The number restriction of *exactly one* will eliminate those pairs whose possible world is w_5.

$$\left\{\begin{array}{l}\langle w_1, A_1\rangle, \langle w_2, A_2\rangle,\\ \langle w_3, A_3\rangle, \langle w_4, A_4\rangle,\\ \langle w_5, A_5\rangle, \langle w_6, A_6\rangle\end{array}\right\}[\mathtt{ExactlyOne}^{\mathtt{x}}(\mathtt{Linguist(x)})(\exists \mathtt{y} \wedge \mathtt{Paper(y)} \wedge \mathtt{Wrote(x,y)})]$$

$$= \{ \langle w, A\rangle \in c'_{\text{pl}} \mid \#(A(\mathrm{x})) = 1 \}$$

$$= \bigcup \left\{\begin{array}{l} \{ \langle w_1, A'_1\rangle \in \{ w_1, A_1 \} [\exists \mathrm{x} \wedge \exists \mathrm{y}] \mid A'(\mathrm{x}) = \text{Andrew and } A'(\mathrm{y}) = p_1 \},\\ \left\{ \langle w_2, A'_2\rangle \in \{ w_2, A_2 \} [\exists \mathrm{x} \wedge \exists \mathrm{y}] \;\middle|\; \begin{array}{l} A'_2(\mathrm{x}) = \text{Andrew and}\\ \text{either } A'_2(\mathrm{y}) = p_1 \text{ or } A'_2(\mathrm{y}) = p_2 \end{array} \right\},\\ \{ \langle w_3, A'_3\rangle \in \{ w_3, A_3 \} [\exists \mathrm{x} \wedge \exists \mathrm{y}] \mid A'_3(\mathrm{x}) = \text{Bill and } A'_3(\mathrm{y}) = p_3 \},\\ \left\{ \langle w_4, A'_4\rangle \in \{ w_4, A_4 \} [\exists \mathrm{x} \wedge \exists \mathrm{y}] \;\middle|\; \begin{array}{l} A'_4(\mathrm{x}) = \text{Benjmain and}\\ \text{either } A'_4(\mathrm{y}) = p_3 \text{ or } A'_4(\mathrm{y}) = p_4 \end{array} \right\} \end{array}\right\}$$

$$= c''_{\text{sg}}$$

$$\left\{\begin{array}{l}\langle w_1, A_1\rangle, \langle w_2, A_2\rangle,\\ \langle w_3, A_3\rangle, \langle w_4, A_4\rangle,\\ \langle w_5, A_5\rangle, \langle w_6, A_6\rangle\end{array}\right\}[\mathtt{ExactlyOne}^{\mathtt{x}}(\mathtt{Linguist(x)})(\exists \mathtt{y} \wedge \mathtt{Papers(y)} \wedge \mathtt{Wrote(x,y)})]$$

$$= \{ \langle w, A\rangle \in c'_{\text{pl}} \mid \#(A(\mathrm{x})) = 1 \}$$

$$= \bigcup \left\{\begin{array}{l} \{ \langle w_1, A'_1\rangle \in \{ w_1, A_1 \} [\exists \mathrm{x} \wedge \exists \mathrm{y}] \mid A'_1(\mathrm{x}) = \text{Andrew and } A'_1(\mathrm{y}) = p_1 \},\\ \left\{ \langle w_2, A'_2\rangle \in \{ w_2, A_2 \} [\exists \mathrm{x} \wedge \exists \mathrm{y}] \;\middle|\; \begin{array}{l} A'_2(\mathrm{x}) = \text{Andrew and}\\ \text{either } A'_2(\mathrm{y}) = p_1 \text{ or } A'_2(\mathrm{y}) = p_2 \text{ or } A'_2(\mathrm{y}) = p_1 \oplus p_2 \end{array} \right\},\\ \{ \langle w_3, A'_3\rangle \in \{ w_3, A_3 \} [\exists \mathrm{x} \wedge \exists \mathrm{y}] \mid A'_3(\mathrm{x}) = \text{Bill and } A'_3(\mathrm{y}) = p_3 \},\\ \left\{ \langle w_4, A'_4\rangle \in \{ w_4, A_4 \} [\exists \mathrm{x} \wedge \exists \mathrm{y}] \;\middle|\; \begin{array}{l} A'_4(\mathrm{x}) = \text{Bill and}\\ \text{either } A'_4(\mathrm{y}) = p_3 \text{ or } A'_4(\mathrm{y}) = p_3 \text{ or } A'_4(\mathrm{y}) = p_3 \oplus p_4 \end{array} \right\} \end{array}\right\}$$

$=c''_{\text{pl}}$

Since $c''_{\text{sg}} \subset, c''_{\text{pl}}$, a scalar implicature is generated, yielding the following set:

$$\{\,\langle w_2, A'_2\rangle \in \{\,w_2, A_2\,\}\,[\exists \mathrm{x} \wedge \exists \mathrm{y}] \mid A'_2(\mathrm{x}) = \text{Andrew and } A'_2(\mathrm{y}) = p_1 \oplus p_2\,\}$$
$$\cup$$
$$\{\,\langle w_4, A'_4\rangle \in \{\,w_4, A_4\,\}\,[\exists \mathrm{x} \wedge \exists \mathrm{y}] \mid A'_4(\mathrm{x}) = \text{Benjmain and } A'_4(\mathrm{y}) = p_3 \oplus p_4\,\}$$

So in the end, only w_2 and w_4 survived, as desired. Furthermore, this accounts for cross-sentential anaphora naturally:

(36) He$_{\mathrm{x}}$ submitted them$_{\mathrm{y}}$ to journals.

5.4 'Everyone'

The same mechanism makes good predictions for other quantificational contexts.

(37) a. Everyone$^{\mathrm{x}}$ wrote papers$^{\mathrm{y}}$. $\rightsquigarrow \mathtt{Everyone}^{\mathrm{x}}(\exists \mathrm{y} \wedge \mathtt{Papers}(\mathrm{y}) \wedge \mathtt{Wrote}(\mathrm{x}, \mathrm{y}))$
b. Everyone$^{\mathrm{x}}$ wrote a$^{\mathrm{y}}$ paper. $\rightsquigarrow \mathtt{Everyone}^{\mathrm{x}}(\exists \mathrm{y} \wedge \mathtt{Paper}(\mathrm{y}) \wedge \mathtt{Wrote}(\mathrm{x}, \mathrm{y}))$

The predicted scalar inference is that at least one person wrote multiple papers.
This is because the singular version (37b) will produce a set of pairs $\langle w, A\rangle$ such that for each $a \in A$, $a(\mathrm{y})$ is an atomic entity that is a paper in w, and $a(\mathrm{x})$ is its author in w, and $A(\mathrm{x})$ is the plurality consisting of all the (relevant) people.
The pairs resulting from (37a) will contain in addition to these pairs, those pairs $\langle w, A\rangle$ such that for some $a \in A$, $a(\mathrm{y})$ is a plural entity that is a paper in w. And only these pairs remain after computing the scalar implicature.
Here are the details. First, we need the distributivity operator, which creates quantificational dependency.[8]

Definition 11. (Distributivity Operator)

$$c[\mathrm{D}_{\mathrm{x}}(\varphi)] := \left\{ \langle w, A'\rangle \,\middle|\, \begin{array}{l} \text{for some } \langle w, A\rangle \in c,\, A(\mathrm{x}) = A'(\mathrm{x}) \text{ and} \\ \text{for each } e \sqsubseteq_a A(\mathrm{x}), \langle w, A'{\restriction_{\mathrm{x}\mapsto e}}\rangle \in \{\,\langle w, A{\restriction_{\mathrm{x}\mapsto e}}\rangle\,\}\,[\varphi] \end{array} \right\}$$
$$e \sqsubseteq_a E :\Leftrightarrow e \sqsubseteq E \text{ and } e \text{ is atomic}$$
$$A{\restriction_{x\mapsto e}} := \{\, a \in A \mid a(\mathrm{x}) = e\,\}$$

(38) $c[\mathtt{Everyone}^{\mathrm{x}}(\varphi)] = \{\,\langle w, A\rangle \in c[\mathrm{M}^{\mathrm{x}}(\mathrm{D}_{\mathrm{x}}(\mathtt{Human}(\mathrm{x}) \wedge \varphi))] \mid A(\mathrm{x}) = \bigoplus\{\, e \in D \mid e \text{ is a human in } w\,\}\,\}$

(39) Consider the following worlds, each with three humans, Andrew, Bill, and Chris.

- w_1: Andrew wrote exactly one paper, p_1, Bill wrote exactly one paper, p_2, and Chris wrote exactly one paper, p_3.
- w_2: Andrew wrote exactly two papers, p_1 and q_1, Bill wrote exactly two papers, p_2 and q_2, and Chris wrote exactly two papers, p_3 and q_3.
- w_3: Andrew wrote exactly two papers, p_1 and q_1, Bill wrote exactly one paper p_2 and Chris wrote exactly one paper p_3.
- w_4: Andrew wrote exactly two papers, p_1 and q_1, Bill and Chris wrote no papers.
- w_5: No one wrote any paper.

[8] We actually have to deal with partiality more carefully in the general case, but to keep the exposition simple, I'll ignore it (this is fine because we are only talking about cases involving random assignment). See Van den Berg (1996) and Nouwen (2003b) in particular. See also Nouwen (2003b) and Nouwen (2007) for an undergeneration problem of this system and a solution to it.

$$\left\{\begin{array}{l}\langle w_1, A_1\rangle, \langle w_2, A_2\rangle, \langle w_3, A_3\rangle \\ \langle w_4, A_4\rangle, \langle w_5, A_5\rangle\end{array}\right\}[\mathtt{M}^{\mathtt{x}}(\mathtt{D}_{\mathtt{x}}(\mathtt{Human(x)} \wedge \exists\mathtt{y} \wedge \mathtt{Paper(y)} \wedge \mathtt{Wrote(x,y)}))] = c'_{\text{sg}}$$

$$\left\{\begin{array}{l}\langle w_1, A_1\rangle, \langle w_2, A_2\rangle, \langle w_3, A_3\rangle \\ \langle w_4, A_4\rangle, \langle w_5, A_5\rangle\end{array}\right\}[\mathtt{M}^{\mathtt{x}}(\mathtt{D}_{\mathtt{x}}(\mathtt{Human(x)} \wedge \exists\mathtt{y} \wedge \mathtt{Paper(y)} \wedge \mathtt{Wrote(x,y)}))] = c'_{\text{pl}}$$

$\mathbf{W}_{c'_{\text{sg}}} = \mathbf{W}_{c'_{\text{pl}}} = \{\, w_1, w_2, w_3, w_4 \,\}$. Note that w_4 is not excluded at this point.

- For each $\langle w', A'\rangle \in c'_{\text{sg}}$, for each $a' \in A'$, $a(\mathrm{y})$ is an atomic entity that is a paper in w', and $a(\mathrm{x})$ is its author in w'.
- For each $\langle w', A'\rangle \in c'_{\text{pl}}$, for each $a' \in A'$, each atomic part of $a(\mathrm{y})$ is a paper in w', and $a(\mathrm{x})$ is the author of the paper or papers in w'.
- Fro each $\langle w', A'\rangle \in c'_{\text{sg/pl}}$, $A'(\mathrm{x}) =$ Chris if $w' = w_4$ and $A'(\mathrm{x}) =$ Andrew $\oplus$ Bill $\oplus$ Chris, if otherwise.

$$\left\{\begin{array}{l}\langle w_1, A_1\rangle, \langle w_2, A_2\rangle, \langle w_3, A_3\rangle \\ \langle w_4, A_4\rangle, \langle w_5, A_5\rangle\end{array}\right\}[\mathtt{Everyone}^{\mathtt{x}}(\exists\mathtt{y} \wedge \mathtt{Paper(y)} \wedge \mathtt{Wrote(x,y)})]$$

$$= \{\, \langle w', A'\rangle \in c'_{\text{pl}} \mid A'(\mathrm{x}) = \bigoplus \{\, e \in D \mid e \text{ is a human in } w \,\}\,\}$$

$$= \bigcup \left\{ \begin{array}{l}
\left\{ \langle w_1, A'_1\rangle \in \{\langle w_1, A_1\rangle\}[\exists x \wedge \exists y] \;\middle|\; \begin{array}{l} A'_1(\mathrm{x}) = \text{Andrew} \oplus \text{Bill} \oplus \text{Chris and} \\ A'_1(\mathrm{y}) = p_1 \oplus p_2 \oplus p_3 \text{ and for each } a' \in A'_1, \\ a'(\mathrm{x}) = \text{Andrew iff } a'(\mathrm{y}) = p_1, \text{ and} \\ a'(\mathrm{x}) = \text{Bill iff } a'(\mathrm{y}) = p_2, \text{ and} \\ a'(\mathrm{x}) = \text{Chris iff } a'(\mathrm{y}) = p_3 \end{array} \right\}, \\
\left\{ \langle w_2, A'_2\rangle \in \{\langle w_2, A_2\rangle\}[\exists x \wedge \exists y] \;\middle|\; \begin{array}{l} A'_2(\mathrm{x}) = \text{Andrew} \oplus \text{Bill} \oplus \text{Chris and} \\ A'_2(\mathrm{y}) = p_1 \oplus p_2 \oplus p_3 \oplus q_1 \oplus q_2 \oplus q_3 \text{ and} \\ \text{for each } a' \in A'_2, \\ a'(\mathrm{x}) = \text{Andrew iff } a'(\mathrm{y}) = p_1 \text{ or } a'(\mathrm{y}) = q_1, \text{ and} \\ a'(\mathrm{x}) = \text{Bill iff } a'(\mathrm{y}) = p_2 \text{ or } a'(\mathrm{y}) = q_2, \text{ and} \\ a'(\mathrm{x}) = \text{Chris iff } a'(\mathrm{y}) = p_3 \text{ or } a'(\mathrm{y}) = q_3 \end{array} \right\}, \\
\left\{ \langle w_3, A'_3\rangle \in \{\langle w_3, A_3\rangle\}[\exists x \wedge \exists y] \;\middle|\; \begin{array}{l} A'_3(\mathrm{x}) = \text{Andrew} \oplus \text{Bill} \oplus \text{Chris and} \\ A'_3(\mathrm{y}) = p_1 \oplus p_2 \oplus p_3 \oplus q_1 \text{ and} \\ \text{for each } a' \in A'_3, \\ a'(\mathrm{x}) = \text{Andrew iff } a'(\mathrm{y}) = p_1 \text{ or } a'(\mathrm{y}) = q_1, \text{ and} \\ a'(\mathrm{x}) = \text{Bill iff } a'(\mathrm{y}) = p_2, \text{ and} \\ a'(\mathrm{x}) = \text{Chris iff } a'(\mathrm{y}) = p_3 \end{array} \right\}
\end{array} \right\}$$

$$= c''_{\text{sg}}$$

$$\left\{\begin{array}{l}\langle w_1, A_1\rangle, \langle w_2, A_2\rangle, \langle w_3, A_3\rangle \\ \langle w_4, A_4\rangle, \langle w_5, A_5\rangle\end{array}\right\}[\mathtt{Everyone}^{\mathtt{x}}(\exists\mathtt{y} \wedge \mathtt{Paper(y)} \wedge \mathtt{Wrote(x,y)})]$$

$$= \{\, \langle w', A'\rangle \in c'_{\text{pl}} \mid A'(\mathrm{x}) = \bigoplus \{\, e \in D \mid e \text{ is a human in } w \,\}\,\}$$

$$
=\bigcup \left\{ \begin{array}{l}
\left\{ \langle w_1, A'_1\rangle \in \{\langle w_1, A_1\rangle\}[\exists x \wedge \exists y] \;\middle|\; \begin{array}{l}
A'_1(\mathrm{x}) = \text{Andrew} \oplus \text{Bill} \oplus \text{Chris and} \\
A'_1(\mathrm{y}) = p_1 \oplus p_2 \oplus p_3 \text{ and for each } a' \in A'_1, \\
a'(\mathrm{x}) = \text{Andrew iff } a'(\mathrm{y}) = p_1, \text{ and} \\
a'(\mathrm{x}) = \text{Bill iff } a'(\mathrm{y}) = p_2, \text{ and} \\
a'(\mathrm{x}) = \text{Chris iff } a'(\mathrm{y}) = p_3
\end{array} \right\}, \\
\left\{ \langle w_2, A'_2\rangle \in \{\langle w_2, A_2\rangle\}[\exists x \wedge \exists y] \;\middle|\; \begin{array}{l}
A'_2(\mathrm{x}) = \text{Andrew} \oplus \text{Bill} \oplus \text{Chris and} \\
A'_2(\mathrm{y}) = p_1 \oplus p_2 \oplus p_3 \oplus q_1 \oplus q_2 \oplus q_3 \text{ and} \\
\text{for each } a' \in A'_2, \\
a'(\mathrm{x}) = \text{Andrew iff } a'(\mathrm{y}) = p_1 \text{ or } a'(\mathrm{y}) = q_1 \\
\qquad \text{or } a'(\mathrm{y}) = p_1 \oplus q_1, \text{ and} \\
a'(\mathrm{x}) = \text{Bill iff } a'(\mathrm{y}) = p_2 \text{ or } a'(\mathrm{y}) = q_2 \\
\qquad \text{or } a'(\mathrm{y}) = p_2 \oplus q_2, \text{ and} \\
a'(\mathrm{x}) = \text{Chris iff } a'(\mathrm{y}) = p_3 \text{ or } a'(\mathrm{y}) = q_3 \\
\qquad \text{or } a'(\mathrm{y}) = p_3 \oplus q_3
\end{array} \right\}, \\
\left\{ \langle w_3, A'_3\rangle \in \{\langle w_3, A_3\rangle\}[\exists x \wedge \exists y] \;\middle|\; \begin{array}{l}
A'_3(\mathrm{x}) = \text{Andrew} \oplus \text{Bill} \oplus \text{Chris and} \\
A'_3(\mathrm{y}) = p_1 \oplus p_2 \oplus p_3 \oplus q_1 \text{ and} \\
\text{for each } a' \in A'_3, \\
a'(\mathrm{x}) = \text{Andrew iff } a'(\mathrm{y}) = p_1 \text{ or } a'(\mathrm{y}) = q_1 \\
\qquad \text{or } a'(\mathrm{y}) = p_1 \oplus q_1, \text{ and} \\
a'(\mathrm{x}) = \text{Bill iff } a'(\mathrm{y}) = p_2, \text{ and} \\
a'(\mathrm{x}) = \text{Chris iff } a'(\mathrm{y}) = p_3
\end{array} \right\}
\end{array} \right\}
$$

$$=c''_{\text{pl}}$$

As before, $c''_{\text{sg}} \subset c''_{\text{pl}}$, and after computing the scalar implicature, we will get a subset of c''_{pl} such that for each $\langle w'', A''\rangle$ in this set, there is at least one $a'' \in A''$ such that $a''(\mathrm{y})$ is a plural entity. This means that w_1 is no longer in this set.

Again, we don't need local computation of scalar implicatures, or higher order implicatures.

also, for Spector (2007), the default reading is what looks like the embedded scalar reading, i.e. everyone wrote multiple papers.[9] This is because the crucial alternative with a scalar implicature means (40).

(40) Everyone wrote at least one paper and not every wrote multiple papers.

5.5 Bonus: *Every* + Disjunction

The current account derives Crnič, Chemla & Fox's (2015) observation about disjunction under a universal quantifier.

(41) Everyone speaks French or German.

According to the 'standard view', this sentence has as scalar implicatures the negations of (42).

(42) a. Everyone speaks French.
b. Everyone speaks German.
c. Everyone speaks both French and German.

Crnič et al. (2015) point out that this prediction is too strong, because (41) is judged as compatible with (42a) or (42b) (though not both at the same time). In other words, we do not want to have the negations of (42a) and (42b) as scalar implicatures.

Crnič, Chemla & Fox's (2015) propose that the relevant scalar implicatures are locally exhaustified versions of (42a) and (42b):

[9] Similarly for Križ (2017). Many thanks to Manuel Križ (p.c.) for discussion on this.

(43) a. Everyone speaks French but not German.
b. Everyone speaks German but not French.

Under the present account, local exhaustification is unnecessary. First, we can treat *or* as an existential quantifier, which makes sense since it feeds anaphora:

(44) Bill speaks French $\text{or}^{\mathtt{x}}$ German (but I don't remember which). He learned $\text{it}_{\mathtt{x}}$ at school.

Assume that we have the following alternatives:

(45) $\text{Everyone}^{\mathtt{x}}$ speaks French $\text{or}^{\mathtt{y}}$ German.
a. $\text{Everyone}^{\mathtt{x}}$ speaks $\text{French}^{\mathtt{y}}$.
b. $\text{Everyone}^{\mathtt{x}}$ speaks $\text{German}^{\mathtt{y}}$.
c. $\text{Everyone}^{\mathtt{x}}$ speaks French $\text{and}^{\mathtt{y}}$ German.

In the standard view, the scalar implicatures derived from (45a) and (45b) are too strong, but in the current view, we are not excluding all possible worlds where everyone speaks French or where everyone speaks German, but only those pairs consisting of one of such possible worlds and an assignment A such that $A(\mathtt{x})$ is French or $A(\mathtt{x})$ is German. Therefore, we keep those possible worlds paired with an assignment A that is not uniform with respect to $\mathtt{x}$, that is, for some $a \in A$, $a(\mathtt{x})$ is French, and for other $a \in A$, $a(\mathtt{x})$ is German.
More concretely:

(46) Assume the following possible worlds:
- w_1: Everyone speaks French and no one speaks German.
- w_2: Everyone speaks German and no one speaks French.
- w_3: Some speak French but not German and some speak German and not French.
- w_4: Everyone speaks French and some speak German.
- w_5: Everyone speaks German and no one speaks French.
- w_6: Everyone speaks both French and German.

$$\left\{\begin{array}{l}\langle w_1, A_1\rangle, \langle w_2, A_2\rangle, \langle w_3, A_3\rangle \\ \langle w_4, A_4\rangle, \langle w_5, A_5\rangle, \langle w_6, A_6\rangle\end{array}\right\}[\mathtt{M}^{\mathtt{x}}(\mathtt{D}_{\mathtt{x}}(\mathtt{Human(x)} \wedge \exists\mathtt{y} \wedge \mathtt{French(y)} \wedge \mathtt{Speak(y)}))] = c'_{\vee}$$
$$\left\{\begin{array}{l}\langle w_1, A_1\rangle, \langle w_2, A_2\rangle, \langle w_3, A_3\rangle \\ \langle w_4, A_4\rangle, \langle w_5, A_5\rangle, \langle w_6, A_6\rangle\end{array}\right\}[\mathtt{M}^{\mathtt{x}}(\mathtt{D}_{\mathtt{x}}(\mathtt{Human(x)} \wedge \exists\mathtt{y} \wedge \mathtt{French(y)} \wedge \mathtt{Speak(y)}))] = c'_{F}$$
$$\left\{\begin{array}{l}\langle w_1, A_1\rangle, \langle w_2, A_2\rangle, \langle w_3, A_3\rangle \\ \langle w_4, A_4\rangle, \langle w_5, A_5\rangle, \langle w_6, A_6\rangle\end{array}\right\}[\mathtt{M}^{\mathtt{x}}(\mathtt{D}_{\mathtt{x}}(\mathtt{Human(x)} \wedge \exists\mathtt{y} \wedge \mathtt{German(y)} \wedge \mathtt{Speak(y)}))] = c'_{G}$$
$$\left\{\begin{array}{l}\langle w_1, A_1\rangle, \langle w_2, A_2\rangle, \langle w_3, A_3\rangle \\ \langle w_4, A_4\rangle, \langle w_5, A_5\rangle, \langle w_6, A_6\rangle\end{array}\right\}[\mathtt{M}^{\mathtt{x}}(\mathtt{D}_{\mathtt{x}}(\mathtt{Human(x)} \wedge \exists\mathtt{y} \wedge \mathtt{German(y)} \wedge \mathtt{Speak(y)}))] = c'_{\wedge}$$

Every$^{\mathtt{x}}$ throws away those pairs $\langle w, A\rangle$ such that $A(\mathtt{x})$ is not all people in w.
Crucially, after the scalar implicature, pairs like $\langle w_4, A'_4\rangle$ will remain, where $A'_4(\mathtt{y}) = \text{French} \oplus \text{German}$ but for some $a, b \in A'_4$, $a(\mathtt{y}) = \text{French}$ and $b(\mathtt{y}) = \text{German}$, because such pairs are not generated by the alternatives.
Prediction (to be tested): *it* in the following must vary between French and German (across linguistic subjects).

(47) Everyone speaks French or German. They all learned it at school.

This sentence should be false in the following model with five people:

- Jean and Marie speak French natively, learned German at school.
- Wataru speaks Japanese natively, learned German at school.
- Katie speaks English natively, learned German at school.
- Ivan speaks Russian natively, learned German at school.

vs.

- Jean and Marie speak French natively, learned German at school.
- Wataru speaks Japanese natively, learned French at school.
- Katie speaks English natively, learned German at school.
- Ivan speaks Russian natively, learned French at school.

6 Conclusions and Further Thoughts

What I proposed is a rather conservative account of plurality inferences in the sense that it makes use of two old ideas, the Gricean Maxim of Quantity and dynamic semantics. It explains a lot of data without additional mechanisms such as local scalar implicatures or higher-order implicatures (although I probably need local computation for some data, e.g. embedding under universal quantifiers).

Being a scalar implicature theory, it crucially assumes that the plural is number neutral, but this assumption is not so innocuous. See discussion in Farkas & de Swart (2010), Bale & Khanjian (2014).

The idea of implicatures based on anaphoric potentials might give a nice account of the semantics and pragmatics of items like 'epistemic indefinites' and superlative modifiers.

6.1 Primary vs. Secondary Implicatures

Two types of conversational implicatures are often distinguished, *primary* and *secondary* (Sauerland 2004).

(48) Some of these movies are interesting.
  a. $\neg$*the speaker believes all of these movies are interesting* (Primary)
  b. *the speaker believes* $\neg$*all of these movies are interesting* (Secondary)

Standard Gricean reasoning generates a primary implicature, which may be strengthened to a secondary implicature via additional assumptions, e.g. Opinionatedness.

(49) a. Suppose that the speaker obeys the Maxim of Quantity.
  b. If she is certain that the alternative sentence *All of these movies are interesting* is true, she should have uttered it.
  c. Because she didn't, she is not certain that it is true.

If there's reason to believe that the speaker is opinionated, i.e. she knows that all of the movies are interesting or that not all of the movies are interesting, then it follows from (49b) that the speaker is certain that not all of the movies are interesting.

Under our account of the plurality inference, we can reformulate the reasoning in terms of not only the speaker's propositional beliefs, but referents the speaker believes a variable varies across.

(50) a. Suppose that the speaker obeys the Maxim of Quantity.

b. If she intends to restrict the referents of the variable only to atomic entities, she should have uttered *Andrew wrote a paper*.
c. Because she didn't, she didn't intend to restrict the referents only to atomic entities.

The resulting primary implicature (50b) is weaker than the plurality inference.

To derive the plurality inference, we just need an extra assumption similar to Opinionatedness, e.g. either the speaker believes that the variable should only vary across atomic entities, or the speaker believes that it should only vary across plural entities.

6.2 Definite Plurals

Definite plurals also give rise to plurality inferences.

(51) a. Chris's student is smart.
b. Chris's students are smart.

The two sentences have different presuppositions.

- If Chris is known to have exactly one student, then (51a) and (51b) will mean the same thing.
- Otherwise, (51a) is not usable. So (51b) should be the only option, and should not have a plurality inference.

Sauerland (2003) derives the plurality inferences of plural definites as **anti-presuppositions**. I could adopt this analysis cases like (51).

Mayr (2015) observes, however, that the anti-presuppositional account wrongly predicts definite plurals to be felicitous in contexts where the exact number is not known. His main data is the following:[10]

(52) *Context: It is common belief that Paul either wrote exactly one song or several songs.*
a. #The song is good.
b. #The songs are good. (Mayr 2015:211)

Mayr (2015) proposes instead that they should be accounted for by NP-level embedded scalar implicatures, and always have plurality inferences.

But do definite plurals always have plurality inferences?

(53) I've never met a female Japanese philosopher or read her papers.

Speculation: The anomaly of (52) might be due to something about restrictions on when the opinionatedness assumption holds and when it does not?

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[10] His other data involve dynamic binding, and I'm not sure whether they should be understood in the same way, although they are certainly problematic for the simplest version of the anti-presupposition idea.

(i) a. #If Paul wrote either several songs or just one, the new song is good.
b. #If Paul wrote either several songs or just one, the new songs are good. (Mayr 2015:212)

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