Expressing Credences

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Abstract
After presenting a simple expressivist account of reports of probabilistic judgments, I explore a classic problem for it, namely the Frege-Geach problem. I argue that is a problem not just for expressivism, but for any reasonable account of ascriptions of graded judgments. I suggest that the problem can be resolved by appropriately modeling imprecise credences.

I Factualism

One of the most basic questions in the study of meaning is what theoretical tools are best suited to represent linguistic meaning. For declarative sentences, the default view is that their meanings should be captured by propositions, which, for the current purposes, can be represented as sets of possible worlds. I call this view *factualism*. Two caveats on my statement of factualism are in order: a) it abstracts away from context dependency, which needs to be captured by any good theory of meaning, and b) representing propositions as sets of possible worlds is not meant to exclude the possibility that something stronger determines a set of possible worlds, such as a structured proposition. I believe that the issues raised by these caveats are orthogonal to those I address here, and so that abstracting from them is harmless.

Propositions are not adequate tools for representing the meaning of all types of sentences. Interrogative sentences, for instance, do not seem to correspond to propositions (Hamblin, 1958). This should raise little concern for the defenders of factualism, however, as we do not naturally think of questions as expressing propositions. We cannot, for instance, pick out a proposition by saying 'the proposition which car is green.' Serious *non-factualism* treats some seemingly factual bits of discourse as deceptive: where propositions appear to be put forward, something else is actually really happening. The most familiar brand of non-factualism is the tradition of *expressivism* in metaethics. According to the expressivist about ethical discourse, moral statements do not really express propositions but rather express or urge moral reactions. Non-factualism about ethics is counterintuitive: it is natural to think that 'murder is wrong' commits one to the proposition (or the fact) that murder is wrong. The non-factualist denies this platitude.[1]

[1] As with many philosophical views of this sort, non-factualists can resort to a distinction between the language of the theorist and ordinary language. This allows them to grant us platitudes with one hand while taking them away with the other.

I shall not be concerned with ethical discourse here. Rather, I will examine the meaning of sentences expressing probabilistic belief, such as 'It's likely that it's raining'. It is natural to think of such sentences as expressing propositions; and so, as in the case of ethical discourse, non-factualism about probabilistic statements is a radical thesis.

I will examine a simple non-factualist account of the meaning and force of such sentences, mostly modeled on recent work by Seth Yalcin and Eric Swanson. My main aim is to assess and respond to a version of the Frege-Geach problem for this kind of expressivism that has recently been pressed in the literature. I will show that by using a suitably refined treatment of probabilistic belief, particularly of mushy credences, we can respond effectively to the objection.

II Sets of worlds in belief and language

As a preliminary, I briefly review the use of sets of worlds to model linguistic meaning and the objects of beliefs, as well as the use of probability measures to model graded belief or credences. As a simplifying measure, which should not affect the issues under discussion here, I treat the collection of all possible worlds as a finite set, W.

The dominant tradition in the study of natural language semantics models the meaning of sentences (in context) as subsets of W, which we will call propositions (e.g., Montague,1973; Lewis,1970; Stalnaker,1970). Thus, abstracting away from issues of context-dependence, a *semantics* for a language $\boldsymbol{L}$ is a function $[[\cdot]] : \boldsymbol{L} \to P(W)$ that takes us from a sentence (we think of $\boldsymbol{L}$ as a set of sentences) to a proposition, a subset of W, capturing its meaning.[2]

Sets of possible worlds also play a central role in philosophical discussions of knowledge and belief (e.g., Hintikka, 1962). Beliefs can be treated as relations between individuals and propositions, which are, again, sets of possible worlds. With respect to graded beliefs, things are however a bit more complicated. We can think of an agent's total *credal state* as being captured by a probability measure over W. A probability measure is just a function $p : P(W) \to [0,1]$ that has these two properties: a) $p(W) = 1$ and b) for disjoint A, $B \subseteq W$, $p(A) + p(B) = p(A \cap B)$. An agent's level of credence in any given proposition is whatever number is assigned to it by the probability measure representing his credal state.

III Probability talk

The uses of sets of worlds reviewed above leave us with a simple and elegant picture of meaning and intentionality: sentences express propositions, beliefs are relations to propositions, and credal states are measures over the field of propositions. A worry emerges, however, when we consider sentences that express credences, such as the following:

(1) It's likely that Switzerland will become a totalitarian state.

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[2] Notation: $\underline{f} : \underline{X} \to \underline{Y}$ indicates a function $\underline{f}$ with domain $\underline{X}$ and range $\underline{Y}$, and $\underline{P}(\underline{X})$ represents the power set of $\underline{X}$, $[[\underline{s}]]$ is result of applying the function $[[\cdot]]$ to $\underline{s}$.

There is, at least, a superficial tension here. From the perspective of philosophy of language, (1) is an ordinary declarative sentence and thus we expect it to express a proposition. From the perspective of formal epistemology, however, things look different. It is certainly true that the embedded sentence, 'Switzerland will become a totalitarian state', expresses a proposition (the set of worlds in which Switzerland becomes a totalitarian state). However, (1) does not correspond to any proposition, any set of worlds; rather it is an expression of a feature of a probability measure over W.[3]

This may only be an apparent tension. We can maintain that a sentence like (1) expresses a proposition like any other sentence by finding an appropriate proposition. One might say, for instance, that (1) expresses the proposition that the utterer's credence function assigns a probability greater than .5 to the set of worlds in which Switzerland will become a totalitarian state. This, then, is just a straightforward factual claim, and can be used to pick out a subset of W, a proposition.

This toy theory faces problems. The content of an utterance by me of (1) does not seem to be the same as the content of a sentence expressing the proposition that I think Switzerland is likely to become a totalitarian state. After all, I can wonder whether (1) is true without simply wondering about what my own mental state is. There are other problems as well, which I do not review here.[4]

Given such difficulties in treating (1) as expressing a proposition, one possibility, which I think is often tacitly accepted in formal epistemology, is to treat sentences such as (1) as distinct in kind from factual sentences. This is the non-factualist proposal about probability talk which is my target in this paper. According to this proposal, factual sentences express facts, and, hence, correspond to propositions; probabilistic sentences do not express facts but rather express features of credal states. Expressing such features of credal states, moreover, is irreducible to expressing any kind of fact.

IV Simple expressivism

I will now give an explicit non-factualist semantics of reports of graded belief. Here I borrow heavily from the work of Yalcin (2005, 2007, 2011, 2010) and Swanson (2006, forthcoming), though my presentation is less attuned to linguistic concerns than theirs. My non-factualist semantics has two components: a formal semantics for a language pairing sentences with semantic values and an account of how to characterize assertions of sentences with those semantic values.

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[3] Swanson (2011) makes this point very clearly. This tension has long been recognized in various guises in formal epistemology (see, e.g., Jeffrey, 1965, ch. 11).

[4] Yalcin (2011) and Swanson (2011) presents many concerns with the program of treating sentences like (1) as propositions.

First, the semantics. Let's start with a factual language $\boldsymbol{L}$ and a semantic function over it, $[[\cdot]]$, pairing elements of $\boldsymbol{L}$, sentences, with subsets of W, propositions. $\boldsymbol{L}$ is the language of straightforward 'facts' about the world, and our semantics assigns each fact a proposition. We understand this language to include sentences such as 'Switzerland will become a totalitarian state', which have as their semantic values the set of worlds in which they are true.

We now expand $\boldsymbol{L}$ by adding a likelihood operator, P, which means roughly 'it's likely that'. The new probabilistic language $\boldsymbol{L}^P$ contains all the elements of $\boldsymbol{L}$ as well as any sentence of the form Ps where $s \in \boldsymbol{L}$.

Expressivism about probability talk can be understood as the view that sentences with the P-operator in front (what I'll call P-sentences) do not express propositions, but rather express credences. One way of modeling this is to treat P-sentences as having as their semantic values not sets of worlds, but rather sets of measure functions. So, we can extend our denotation function $[[\cdot]]$ to cover P-sentences as follows:

(2) For s in $\boldsymbol{L}$, $[[Ps]]$ = the set of probability functions that assign $[[s]]$ a probability greater than .5.

Of course, merely assigning a formal object to a P-sentence does not give us a full theory of what it is to assert such a sentence. What I call *bridging principles* link semantic values to the speech acts associated with the sentences.[5] Here is a rather crude bridging principle covering assertions of P-sentences:

(3) If s is a P-sentence in $\boldsymbol{L}^P$, then an assertion of s is a suggestion that the conversational participants adopt some credal state in $[[s]]$.

This can be contrasted with a standard bridging principle that characterizes assertion of factual sentences:

(4) If s is a factual sentence in $\boldsymbol{L}^P$ then an assertion of s is a suggestion to believe $[[s]]$.

Our non-factualist semantics needs both (3) and (4) for the P-sentences and the factual sentences, respectively. I formulate expressivism this way to make clear that any explicit expressivism must contain not only a semantics for a language, but also bridging principles relating the output of that semantics to the speech act of assertion. Once we have these two elements---the semantics and the bridging principles---we have a full-fledged non-factualist theory.

So far we have constructed a rather crude semantic and pragmatic theory for a syntactically restricted language. The theory is essentially a typed one: there are

[5] Rothschild (2011) discuss bridging principles for expressivism about conditionals. The term is used in a similar way in von Fintel's (2004) discussion of presuppositions.

two kinds of sentences, factual ones and expressivist ones, and different bridging principles apply to each kind. Since expressivism about factual sentences seems unpromising, it would seem that such a typed language should be an expected feature of any expressivist semantics.[6]

V Frege-Geach

There are a variety of worries related to the embedding of *P*-sentences under logical connectives and operators that go under the general label of the Frege-Geach problem (see Schroeder 2008b, for a useful review). Our language $\boldsymbol{L}^P$ includes both factual sentences and *P*-sentences, but syntactically we do not have any means for embedding *P*-sentences under logical operators such as 'and', 'or', and 'not'. For example even a simple conjunction like 'it's likely to be red and it's likely to be big' are not available in $\boldsymbol{L}^P$.

A naïve version of the Frege-Geach problem can be stated as follows: on the expressivist view *P*-sentences express suggestions to conversational participants to do something (namely, adjust their credences). But since there is no natural way of thinking about negations, conjunctions or disjunctions of suggestions---is a disjunction of suggestions a kind of suggestion? if so what kind?---we lack an account of how *P*-sentences behave under logical operators.

This version of the Frege-Geach problem rests on a misunderstanding. The logical operators do not directly act on speech acts, they act on semantic values. In order to handle embeddings of *P*-sentences under the logical operators we need to give a semantics for those operators as they apply to the semantic values of *P*-sentences; we do not need to apply the operators directly to speech acts.

Consider first the factual language $\boldsymbol{L}$. Let us enrich $\boldsymbol{L}$ by allowing conjunctions, negations and disjunctions of sentences in it, yielding a new language $\boldsymbol{L}^O$. The required supplementation of the syntax and semantics of $\boldsymbol{L}$ to yield $\boldsymbol{L}^O$ are familiar from elementary logic and set theory:

SYNTAX: If s and s' are in $\boldsymbol{L}^O$ then so are $s \wedge s'$, $s \vee s'$, and $\neg s$

SEMANTICS: $[[s \wedge s']]=[[s]] \cap [[s']]$, $[[s \vee s']]=[[s]] \cup [[s']]$, and $[[\neg s]]=W \backslash [[s]]$

These definitions give us the standard classical account of the propositional connectives.

In order to isolate out the basic issues to do with the Frege-Geach problem, let's consider a simple extension of $\boldsymbol{L}^O$, which includes *P*-sentences and allows

[6] Both Yalcin (2007) and Swanson (forthcoming) provide a uniform semantic type for all expressions in the language. I would argue, however, that for our purposes these types are effectively disjunctive. Of course, the difference between their way of doing it and mine is almost entirely presentational: a typed language with type-shifting operations—which I would need to handle, e.g., conjunctions of different types—is similar to a non-typed language.

conjunctions, disjunctions and negations of these, but not combinations of *P*-sentences and factual sentences. The supplementary syntactic and semantics rules to take us from $\boldsymbol{L}^{O}$ to $\boldsymbol{L}^{OP}$ are as follows:

> SYNTAX: If *s* is in $\boldsymbol{L}^{O}$ then *Ps* is in $\boldsymbol{L}^{OP}$. If *s* and *s*' in $\boldsymbol{L}^{OP}$ not in $\boldsymbol{L}^{O}$, $s \wedge s'$, then $s \wedge s'$, $s \vee s'$, and $\neg s$ are in $\boldsymbol{L}^{OP}$.
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> SEMANTICS: for *s* and *s'* in $\boldsymbol{L}^{OP}$ and not in $\boldsymbol{L}^{O}$, $[[s \wedge s']] = s \cap s'$, $s \vee s' = [[s]] \cup [[s']]$, and $[[\neg s]] = M \backslash [[s]]$, where *M* is the set of all probability measures over *W*.

This is a basic extension of the definitions of the logical operators to cover *P*-sentences. For $\vee$ and $\wedge$ no changes at all were needed. However, we needed to make reference to *M* rather than *W* to characterize $\neg$.[7]

This formal system is severely limited: it does not allow---either syntactically or semantically---combinations of *P*-sentences and non-*P*-sentences with $\wedge$ or $\vee$. There are reasonable ways of expanding the language to deal with such combination.[8] Instead of looking into these I will examine a problem which arises already for this simple language. Let us take a concrete example of a disjunction of *P*-sentences:

> (5) Either it's likely Estonia will become a totalitarian state or it is likely Lithuania will become a totalitarian state.

Since words like 'likely' do not seem to behave in a standard way under disjunctions, we need to do a little work to get the right reading of (5). The reading suited for our purpose is what you might call the 'I'm not sure which' reading: the sentence means simply that one of the two Baltic countries is likely to become a totalitarian state.[9]

With our semantics and bridging principles, we have an account of what it is to assert (5). Let *e* be the factual sentence 'Estonia will become a totalitarian state', and *l* be the factual sentence 'Lithuania will become a totalitarian state'. In this case, (5) can be written as $Pe \vee Pl$, a sentence of $\boldsymbol{L}^{OP}$. Using the semantic rules above, the denotation of $Pe \vee Pl$, $[[Pe \vee Pl]]$, will be the set of all probability measures that assign a probability greater than .5 to the proposition expressed by *e* or a probalility greater

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[7] A motivation for treating the logical connectives as 'doing the same thing' with different types of semantic values can be found in the type-shifting literature (e.g. Partee and Rooth, 1983).

[8] Both Yalcin and Swanson allow combinations of *P*-sentences and regular factual sentences. Serious foundational questions arise in deciding how to do this, but I know of no problem in principle.

[9] I use disjunction to simplify the discussion. We can, however, find more natural examples of probability operators scoping under logical connectives using quantifiers. For example, I take it that the following sentence has a narrow scope reading of 'likely':
(i) At most three of these candidates are likely to be hired.
See Swanson (2010) for further discussion and examples of narrow scope uses of epistemic modal expressions.

than .5 to the proposition expressed by *l*. Given (3), we can characterize the speech act associated with this sentence, as a suggestion that the conversational participants adopt credences inside the set [[$Pe \vee Ps$]]. This amounts to a suggestion to either adopt a credence on which it is likely that Estonia will become a totalitarian state or a credence on which it is likely that Lithuania will become a totalitarian state.

Our simple expressivist theory is thus in principle capable of handling embeddings of *P*-sentences under logical operators. However, the account seems to mischaracterize the force of disjunctions such as (5). In fact, an assertion of (5) does not recommend the adoption of credences on which it is likely that Estonia will become a totalitarian state or credences on which it is likely that Lithuania will. It seems to me that you needn't believe either is likely to assert or accept (5).[10]

We thus face the following problem: adopting a classical account of disjunction combined with the expressivist framework leads us to make a bad prediction about the force of disjunctions of *P*-sentences. We might take this as a reason to be sceptical about this brand of non-factualism about *P*-sentences. Or we could, following Swanson (2006), keep this general form of expressivism, but replace classical disjunction in favor of something more complex. I shall argue here that both these responses are unwarranted. All we need to do to solve the problem is to adopt a suitably sophisticated and independently motivated account of imprecise credences. Before getting to that, I will argue in the next section that the problem raised here is not just a problem for expressivism, but moreover for any reasonable semantic account of belief attributions of *P*-sentences.

VI Belief ascriptions and disjunction

Here I want to argue for a simple hypothesis about belief ascriptions of *P*-sentences, which I call *belief transparency*:

> BELIEF TRANSPARENCY If *p* is a *P*-sentence then [[*x* thinks *p*]] is the set of worlds where x's credences are in [[*p*]], where [[*p*]] is as defined above.

Whether or not you accept expressivism, belief transparency is plausible.[11] First, just on intuitive grounds. Consider:

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[10] This point is made by Swanson (forthcoming). Swanson calls the semantic values of *P*-sentences 'constraints' (i.e. they are constraints on what credences are acceptable). He writes:

> But the constraint that should be associated with a disjunction cannot, in general, be the union of the constraints associated with each of the disjunctions disjuncts. For example, a believer may believe a disjunction without believing any of its disjuncts. But if the constraint associated with a disjunction were the union of the constraints associated with its disjuncts, this would be impossible.

Schroeder (2011) presses a similar point against Yalcin's expressivism. In addition, as Schroeder pointed out to me (p.c.) this problem is very similar to the problem with negation for moral expressivism discussed by Schroeder (2008a, 2010).

[11] For an extended defense see Yalcin (2011).

(6) Ingrid thinks it's probably raining.

According to belief transparency is true iff it is likely to be raining according to Ingrid's credences.

Compare this with the most natural factualist account of (6), on which the embedded sentence 'it is probably raining' expresses a proposition. On this view (6) ascribes to Ingrid a belief in some proposition *p*, which is the semantic value of `it's probably raining'. What proposition could serve this role? The only plausible candidate would be the proposition that rain is probable according to Ingrid's credences. And this, in turn, is plausible only if we think that one's probabilistic beliefs are transparent; that is, you think that a proposition *p* is likely in general only if you believe that you think that *p* is likely. In this case, (6) will generally be true iff it's likely to be raining according to Ingrid's credence, roughly capturing the intuitive truth conditions.

Here are two problems with this factualist account of (6): First, if (6) states the proposition that Ingrid believes that rain is likely according to her credences, the truth of (6) depends not directly on Ingrid credences but rather on her beliefs about them. This seems wrong. Second, adapting an argument in Yalcin (2007), if we assume that belief ascriptions of probabilistic beliefs are simply ascriptions of beliefs in some propositions, then we do not have an explanation of the difference in meaning between the following two sentences:

(7) a. Ingrid imagined (it was probably raining but it wasn't raining).
b. Ingrid imagined (she believed it was probably raining but it wasn't raining).

It seems to me that (7-a) is attributing a contradictory imaginative state to Ingrid, while (7-b) is not. It's hard to explain this contrast if imagining that it is probably raining is just imagining of oneself that one's rain is likely according to one's credences. (See Yalcin (2007) for further discussion).

Belief transparency, thus, seems very likely to be true.[12] Belief transparency is a sort of expressivism lite: it's an expressivism for probability statements inside belief ascriptions. This means that it faces the same problem that was raised above concerning expressivism and disjunction. Thus, consider:

(8) Ingrid thinks that either it's likely Estonia will become a totalitarian state or it's likely Lithuania will become a totalitarian state.

[12] You might think this alone supports non-factualism, since belief transparency makes reference to the non-factualist semantics. Belief transparency may give prima facie grounds for adopting non-factualism about probabilistic talk, but notice here that it is possible to give a factualist semantics that can support belief transparency. See Yalcin (2007, 2009) for discussion.

If we apply belief transparency then we must conclude that (8) is true iff Ingrid's credences are high in $[[Pe \vee Pl]]$. The problem is thus just as before: either (9-a) or (9-b) must be true for (8) to be true.

(9) a. Ingrid thinks that it's likely Estonia will become a totalitarian state.
b. Ingrid thinks that it's likely Lithuania will become a totalitarian state.

This is a bad prediction, for the reasons discussed above.

We might try to avoid this problem by restricting belief transparency to simple *P*-sentences and give a different treatment of complex combinations of them. The problem with this suggestion is that it requires that a coherent story be told about the meaning of 'either...or' as used in (8) which yields reasonable truth conditions. Restricted belief transparency tells us that *P*-sentences such as 'it is likely that Estonia will become a totalitarian state' put a constraint on credences when they are embedded in a belief ascription. What do we do with a disjunction of such sentences in a belief ascription, as in (8)? Given that 'or' is classical, the only obvious option would seem to be to make the embedded disjunction express a disjunction of conditions on credences. But to meet the disjunctive conditions you have to meet one of the disjuncts, which was our original problem. Thus, it is not clear what those who accept belief transparency can say about belief ascriptions of disjunctions of *P*-sentences.

VII Mushy credences and probabilistic-belief ascription

I have just argued that the problem with disjunction of *P*-sentences is not a problem with our semantics of *P*-sentences per se, but rather with our account of what it is to be in a given credal state. So far we have assumed that a credal state can be represented by a probability measure. This assumption, however, is itself undermined by the kinds of considerations about disjunctions raised in the previous section. Let us accept that (8) is a coherent ascription of some kind of credal state. The question is now: What credal state or property of credal states corresponds exactly to this description? It is true that credal states that are in $[[Pe \vee Pl]]$ are credal states on which an ascription of (8) is true. But these cannot be the only ones, or we get the bad inference from (8) to (9-a) or (9-b). On the other hand, all the credal states which are not in $[[Pe \vee Pl]]$ are ones where $[[e]]$ and $[[l]]$ are assigned probabilities less than .5. It is not clear how to pick out which of these states are those which are ascribed by (8).

A probability measure assigns, by definition, a perfectly precise probability to every subset of *W*. There are many reasons to be sceptical about using a probability measure to model a credal state. Without going into these in detail, we can simply note that it is plausible that a person--even one, perhaps, who is an ideally rational agent--might not assign a unique probability to some event due to lack of information.[13]

We might, then, want a representation of probabilistic belief that is less fine-grained than probability measures. One obvious approach, common in the literature, is to represent a credal state by a set of probability functions rather than a single function: a credal state does not determine a single probability measure but rather a set of them. I'll call such sets *credal sets*. Where an agent assigns a determinate probability to a proposition, every measure in their credal set assigns that probability to it. A probabilistic claim is true of a credal set just in case it is true on every probability measure in the set. When different measures in a credal set assign different probabilities to a proposition, the credences represented by the set are indeterminate between those values.

If credal states are not well-represented by probability measures, then a good expressivist semantics should not treat *P*-sentences as suggestions to adopt a probability measure, as we did above. Swanson (2006, forthcoming) argues that we should use whichever is our best account of credal states in order to model the semantics of sentences which concern those states. Following this suggestion, we can use credal sets rather than single probability measures to give our semantics of belief.[14]

The necessary revisions to our semantics are minimal. We can keep the same semantics for *P*-sentences (and logical combinations of them). All we need to do is adjust our rule of assertion and our semantics for belief attributions. Our previous bridging principle for assertions of *P*-sentences was as follows:

(10) If *s* is a *P*-sentence then an assertion of *s* is a suggestion that the conversational participants adopt some credence that is a subset of [[*s*]].

The only necessary modification is to take sentences as suggestions to have one's credal state be a subset of the denotation of the *P*-sentence. This goes as follows:

(11) If *s* is a *P*-sentence then an assertion of *s* is a suggestion that the conversational participants adopt some credal set that is a subset of [[*s*]].

So, if someone asserts that it is likely that Switzerland will become a totalitarian state, we now understand this claim as a suggestion to adopt a credal set all members of which assign a probability greater than .5 to the proposition that Switzerland will become a totalitarian state. Intuitively, any such credal set is one in which it is likely that Switzerland will become a totalitarian state.

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[13] Halpern (2003) reviews many of the problems with using a single probability measure to model credal states as well as discussing the major alternatives in the literature.

[14] I should note that this proposal follows the ideas of Yalcin (2005, 2007) who suggests modeling presuppositions by sets of probability functions. It is a natural step to also use such sets for belief states.

We need a similar adjustment to our semantics of belief attributions:

(12) If *s* is a *P*-sentence, [[*x* thinks *s*]] is the set of worlds in which *x*'s credal set is a subset of [[*s*]].

This intuitively gets us the right conditions for belief in simple *P*-sentences.

It is an immediate consequence of this revision of our version of expressivism that the problem with disjunction goes away. We can see this for the case of an assertion of (5). This now only counts as a suggestion to make one's credal set include only functions that assign > .5 to the proposition that Lithuania will become a totalitarian state or to the proposition that Estonia will become a totalitarian state. Such a credal set, however, need not be one on which either it is likely that Lithuania will become a totalitarian state or it is likely that Estonia will become a totalitarian state. The problem with belief attributions mentioned in the previous section also goes away for parallel reasons.[15]

So, the simple expressivism I have presented is robust against the Frege-Geach problem with disjunction once we model credal states with credal sets, and adjust our bridging principles and semantics of belief accordingly. This basic approach can easily be extended to treatment of other logical operators such as negation and the quantifiers. In all cases, we can treat the operators as classical.[16]

VIII Factualism regained?

Before closing, I will make some remarks on the question of the extent to which the view just outlined is a non-factualist view. What we have is a view in which we give different types of semantic values to *P*-sentences and to factual sentences. But doing this just raises the question of what really (besides the names) makes one any less factual than the other.

The two types of objects in play in the semantics (credal sets and sets of worlds) have many formal parallels. In each case, belief in a sentence is a matter of whether one's mental state (either a set of worlds believed, or a credal set) is a super set of the semantic value of the sentence. Likewise, entailment relations between sentences can, in each case, be characterized by the subset relation.

We might think that what makes the treatment of *P*-sentences here non-factualist has to do with the need for the special bridging principle (11) above. Perhaps, then, the non-factualism of this semantics is secured by its different

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[15] I should note that the proposal in this section to relate expressions of graded belief to mental states modelled by sets of probability functions relates closely to Malte Willer's (2011) proposal to relate epistemic modals to mental states modeled by sets of sets of worlds. (I am grateful to Mark Schroeder for drawing my attention to this.)
[16] There is an important qualification here. Klinedinst and Rothschild (forthcoming) discuss some way in which the connectives when combined with modals and probability operators are not quite classical.

bridging principles for assertion of factual sentences and *P*-sentences. Factualists sentences recommend belief in a proposition; *P*-sentences recommend conforming one's probabilistic beliefs to a certain condition. However, once we accept the semantics of belief sentences themselves in terms of credal sets and sets of possible worlds, we can unify these two bridging principles into one:

(13) An assertion of *s* is a recommendation to believe [[*s*]].

So, we cannot rely on the disunity of the characterization of assertion as grounds for classifying the semantics as non-factualist. Of course we could claim that the belief in a denotation of a *P*-sentence (represented by a credal set) is not a genuine belief in a proposition, while belief in a denotation of a factual set (a set of possible worlds) is a genuine one. But this simply begs the question.

The claim that the sort of semantic theory presented here should not be viewed as a form of non-factualism dovetails with Moss's (2011) discussion of knowledge of likelihood. She notes that knowledge of *P*-sentences functions in much the same way as knowledge of factual sentences. We can know that something is likely, not merely believe it. Such considerations might push us to accept that both credal sets and sets of worlds can serve as representation of some sort of facts.[17]

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