# Epistemic contradictions: why idempotence is hygienic

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## 1 Introduction

## 2 Dynamic view of modality

Veltman's default semantics:

$c[A] = \{w \in c : A \text{ is true in } w\}$
$c[\neg\phi] = \{w \in c : w \notin c[\phi]\} = c \backslash c[\phi]$
$c[\phi \wedge \psi] = c[\phi] \cap c[\psi]$[1]
$c[\Diamond\phi] = \{w \in c : c[\phi] \neq \emptyset\}$

Motivation for this was entirely difference between these two dialogues:

(1) The keys might be in the car . . . they aren't.

(2) The keys aren't in the car . . . they might be.

Not it seems a very good motivation. However, literature on epistemic modals generally supported some kind of non-standard semantics.

## 3 Yalcin's Epistemic Contradiction and Dynamic Sematnics

Yalcin's [2007] observation that 'it's raining but it might not be raining' exhibits a kind of incoherence beyond Moorean paradox:

(3) a. ?Suppose it's raining but it might not be raining.

b. Suppose it's raining but you'd don't know it's raining.

Hard to give a pragmatic explanation of this on standard semantics of epistemic modals.[2] So, a strong argument for a non-classical semantics for any natural language construction. (Much stronger than arguments based on order which seem dubious.)

## 4 Dynamic treatment of epistemic contradictions

We might first want to look at $c[A \wedge \Diamond\neg A]$. First, what is the meaning of this?

Well, it takes a context: and returns $\emptyset$ if the context has no worlds in which $A$ is false, otherwise it takes out all the not-$A$ worlds. So it's a long winded way of asserting $A$ and making sure the assertion is non-vacuous in the context.

But why is this bad, then?

Think about belief in a dynamic context. You believe a CCP $\phi$ just in case the worlds compatible with your belief, $b$, are a fixed point of $\phi$. So $b[\phi] = b$.

If you have to believe what you assert then we have an explanation of why dynamic conjunctions are unassertable.

## 5 Sidenote: dynamic conjunction and explanation of epistemic contradictions

This is a confusing diversion which is left as an exercise for the audience.

[1] Interestingly, Veltman did not use the standard 'dynamic' conjunction, $c[\phi \wedge \psi] = c[\phi][\psi]$. To be discussed.

[2] See the contortionist act of Dorr and Hawthorne [2014] for evidence of this.

# 6 Negations of epistemic contradictions $\neq$ tautologies

Pure example:

(4) It's not the case that (it's raining and it might not be raining).

Parsable example:

(5) He doesn't think it's raining but it might not be raining.

Note however: $c[\neg(\Diamond\neg A \wedge A)]$ = either $c[\neg A]$ if $c[\neg A] \neq \emptyset$, or $c$ otherwise.

So predictions: from semantics. (4) can be a long-winded assertion of $\neg A$, and (5) attributes to John a state that is a fixed point of $[\neg(\Diamond\neg A \wedge A)]$ which is just that he either believes $A$ or $\neg A$!!!!

Regardless of what exactly these sentences means this is not it. Probably, best to treat $\neg(A \wedge \Diamond\neg A)$ as a tautology.

Another problem: $(\Diamond\neg A \wedge A) \rightarrow B$. Dynamic test semantics for conditionals: $c[\phi \rightarrow \psi] = c$ if $c[\phi][\psi] = c[\phi]$, $\emptyset$ otherwise.

# 7 Fix

There's a problem and there's a fix. Note first as a background (and advertisement) a very general characterizing result about dynamic semantics proved by Rothschild and Yalcin [2012]. We showed that a semantic system is truly dynamic (in the sense of not being isomorphic to a static, Stalnaker-style update system) iff it is not both idempotent and commutative. For all $c$ and $\phi$ and $\psi$: idempotence is $c[\phi] = c[\phi][\phi]$, commutativity $c[\phi][\psi] = c[\psi][\phi]$.

We argue there that idempotence is a very natural property of natural language, and it is not the usual motivation or dynamic semantics. Note, however, that Veltman's semantics is not idempotent:[3]

$c[A \wedge \Diamond\neg A] \neq c[A \wedge \Diamond\neg A][A \wedge \Diamond\neg A]$

[3]Thanks to Thony Gillies for pointing this out to me ages ago.

Also not commutative: $c[A][\Diamond\neg A] \neq c[\Diamond\neg A][A]$.

What we see now is *exactly what makes Veltman's/Yalcin's semantics non-idempotent also produces problematic predictions.*[4]

This leads us to the hypothesis that non-idempotence is *not* a desirable feature of dynamic semantics and should be eliminated. In particular, the idea is that what is not allowed at the sentential level (non-idempotent updates) is also not allowed at the intra-sentential level. So one suggestion is to modify semantics to enforce a kind of idempotence. This gives us a fixed-point dynamic update system (at the intrasentential level).

For any operation [] on a contexts we define $c[]^*$ to be $c'$ such that there exists an $n$ and for all $i > n$ $i$ applications of [] on $c$ equals $c'$, if there is no such $n$, $c[]^* = 0$.[5]

New semantics:[6]

$c[A] = \{w \in c : A \text{ is true in } w\}$
$c[\neg\phi] = \{w \in c : w \notin c[\phi]^*\} = c \backslash c[\phi]^*$
$c[\phi \wedge \psi] = c[\phi]^* \cap c[\psi]^*$
$c[\Diamond\phi] = \{w \in c : c[\phi]^* \neq \emptyset\}$

We now think of the update of $c$ with $\phi$ as $c[\phi]^*$ (which is anyway at the sentential level what we were assuming all along to explain the badness of epistemic contradictions).

Now, note the following $c[A \wedge \Diamond\neg A]* = \emptyset$, $c[\neg(A \wedge \Diamond\neg A)]* = c$.

# 8 Speculations

Suppose De Morgan's law defines disjunctions: $\phi \vee \psi = \neg(\neg\phi \wedge \neg\psi)$. Then, $\neg(A \wedge \Diamond\neg A) = \neg A \vee \neg\Diamond\neg A = \neg A \vee \Box A$. So, amazingly, we get for free the tautologous status of (6).

(6) Either it's not raining or it must be.

[4]Note describing Yalcin's semantics as non-idempotent is a bit of a stretch, but given basic equivalence, I hope unproblematic.

[5]Actually it's easily provable that there will always be such an $n$ for this semantics.

[6]Thanks to Wes Holliday for suggesting this way of formulating a general idempotence requirement across a semantics.

(We no longer need explanation in terms of dynamic connectives as pursued in Klinedinst and Rothschild [2012]. Maybe?) Maybe we can wedge away dynamic semantics from the dynamic connectives, taking order effects as processing not semantics.

# References


Cian Dorr and John Hawthorne. Embedding epistemic modals. *Mind*, 122: 867–913, 2014.

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