

Do Indicative Conditionals Express Propositions?

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1 Two Views of Meaning

In this paper I will focus on the question of what theoretical object we should use to represent the meaning of certain sentences. In particular, I will focus on the question of whether we should use propositions to represent the meaning of indicative conditionals. Before getting into this question I'll try to locate it in the broader scheme of things.

In recent years, the so-called semantic/pragmatics debate has focused on the gap between sentence meaning and speaker meaning. Generally, participants in that debate take for granted that we use sentences to express propositions, objects that at the minimum determine a set of truth-conditions. The semantics/pragmatics debate focuses on *how* sentences are used to express propositions. However, the question I will focus on is not *how* sentences come to express whatever they do express, but rather *what*, at the end of the day, they do express.

There is plenty of controversy about whether sentences really do express propositions, even after pragmatic enrichment. There are some philosophical traditions that view talk of truth conditions and propositions as generally suspect.¹ The considerations that push people to this view tend to be programmatic philosophical ones. I am not going to discuss this sort of *blanket* anti-propositionalism.

However, even outside of these traditions there are some who think that, while sentences can in general express propositions, as it happens some sentences of our language do not do so. There are usually two different sorts of motivations for this view. On the one hand, you might think that the kind of facts a given part of language aims to express simply don't exist. For instance, many philosophers worry about whether moral claims can really, properly speaking, be true or false. This worry then extends to a worry about

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¹Many understand the late Wittgenstein this way. More recent pragmatists such as Robert Brandom would fall into this category, too.

whether sentences expressing moral claims could possibly be used to express propositions. I'll call anti-propositionalism motivated this way *domain-driven*. Ultimately this sort of anti-propositionalism seems to rely not on considerations about how our language works, but rather on considerations about what sort of facts are available for us to express.

Another, more linguistic, variant of anti-propositionalism also exists. On this view, there are certain aspects of our language that are simply, on their own terms, better characterized by rules that do not require a pairing of propositions with sentences. That is, aspects of the very rules governing our linguistic usage motivate the claim that certain sentences don't express propositions. This anti-propositionalism is significant for linguists because it rests not on abstract claims about the nature of meaning, but rather on the contingent facts about our language, or at least some parts of our language.

Obviously, it's not always going to be clear whether a given sort of anti-propositionalism is really domain-driven or linguistically-driven—and all varieties of anti-propositionalism depend both on what the linguistic rules are and on what the world is like. But intuitively it seems like it is not the grammar of a word like "good" or "right" that makes people think that sentences with these words don't express propositions. It isn't, in fact, obvious that the grammar of "good" is much different from that of "tall." On the other hand, I'll suggest that with epistemic modals and conditionals, worries about whether they express propositions are much more closely linked to the basic linguistic rules governing them.²

Now, some see in anti-propositionalism an anti-theoretical or an anti-formal bent. However, recent work has dispelled the Wittgensteinian air from the position. One can be an anti-propositionalist about some parts of language and still endorse traditional formal methods for studying language. (Of course, the relevant question is not the formal methods themselves, but rather the precision of claims one is making about language: it just happens that formal methods facilitate precision.) There is a natural framework for viewing meaning non-propositionally that can easily be made rigorous and systematic.³ In this framework, roughly speaking, we view sentences not as expressing propositions but rather as directly expressing properties of attitudes. I'll call this the *attitude-view* of semantics, and as we shall see, it's naturally thought of, in more traditional terms, as a form of *expressivism*.

One way to see the difference between the attitude-view of semantics and the propositional view is in their differing expressive power. Asserting propositions may be seen as *one* way of expressing properties of attitudes. The property of attitudes expressed when I assert, say, "John went to the bank," is simply the property of believing the proposition that

²I am relying on a conception of linguistic rules governing expressions that doesn't simply amount to all the norms governing their use. Obviously, it is all of the norms governing our use of "good" that would make one doubt that certain expressions with "good" express propositions. Nonetheless, in general, those working in linguistic semantics don't think of the question of exactly which property is picked out by an expression to be a linguistic question, so the norms governing our use of the word "good" need not all be linguistically represented. Rather for the purposes of linguistics it might just be that "good" encodes a gradable adjective, not very different from "tall".

³The work of Yalcin (2007) and Swanson (2006) are good examples.

John went to the bank. So every propositional assertion can, in this way, be described as a property of an attitude.

However, it is not in general true that every property of an attitude can be understood as a propositional assertion. Consider, for example, the property of not ruling out the proposition p as a doxastic possibility. In general, there is no single proposition one can accept such that one doesn't rule out p if and only if one accepts that proposition (p itself is too strong). So if our language provides the resources to simply express that we don't rule out p , then it seems we will not be able to understand that bit of language if we confine ourselves to a framework where sentences express propositions.

Indeed, our language does seem to allow us to express that we don't rule out p . We can do this by using epistemic modals such as "might":

(1) It might be dark outside.

On a natural understanding, (1) would seem to express that we do not want to rule out the proposition that it is dark outside. It doesn't appear to express anything more than that. So since there is no proposition whose acceptance amounts to allowing a possibility of darkness outside, we might think that we should model the meaning of "might" by using the attitude view.

The above sketch of an argument for the attitude view is far too quick. After all, we can use certain propositions to express any property of attitudes we want: all we need is to use propositions about our mental states. So, why shouldn't we understand (1) as being a statement about our beliefs? In this way, then, (1) would express a constraint on attitudes by way of expressing a proposition about mental states.

Taking (1) to be a statement about our beliefs is not a very promising strategy. For example, if we negate (1) we get a new sentence which clearly entails that it is not dark outside:

(2) It's not the case that it might be dark outside.

However, if we negate a statement about our beliefs we get no such entailment:

(3) It's not the case that it's compatible with my beliefs that it is dark outside.

All is not lost, however, for the propositional view. If we switch belief with knowledge then this particular problem goes away. We might then think that (1) expresses a proposition that can be roughly paraphrased as follows:

(4) That it is dark outside is compatible with my knowledge.

If we negate this we do get something that entails that it is dark outside, by the factivity of knowledge. Indeed, propositionalism (via attitudes) about epistemic modals is probably the

most common strategy in the semantics and philosophical literature, despite its problems.⁴

In this paper, I do not address propositionalism about epistemic modals, but rather discuss propositionalism about conditionals. The relationship between conditionals and probability creates a particularly pressing problem for propositionalism, one which I think has never been adequately answered by propositionalists. Here I sketch both the problem and a way propositionalists can respond to it. I remain skeptical of propositionalism about indicative conditionals, but I think the worry from conditionals and probability is not insurmountable.

2 The Problem with Conditionals and Probability

Consider these two sentences:

- (5) a. It's likely that the airbag will go off.
b. It's likely that if the car crashes at greater than 35mph, the airbag will go off.

Intuitively, it seems that both sentences are of the form: *it's likely that x*, where, in the first case, $x = \text{the airbag will go off}$ and, in the second case, $x = \text{if the car crashes at greater than 35mph, the airbag will go off}$. Understanding how these sentences get their meaning as a function of the meaning of both the embedding construction *it's likely that ...* and the embedded sentence, x , would seem an easy task. The natural way would be to assume i) that in each case the particular sentence x expresses a proposition, and ii) that sentences of the form *it's likely that x* are true iff x expresses a proposition that has a probability greater than one-half. In other words, in general, *it's likely that x* is true iff $p(x) > .5$. This simple proposal leaves various questions unanswered. For instance, what does it mean for x to have a probability greater than one-half? Is it that the speaker has a credal state that assigns a subjective probability of more than one-half to x , or is that there is a more objective or intersubjective probability at stake?⁵ I will put this question aside, though, as a more basic problem confronts the proposal.

The problem goes as follows: Suppose we understand (5-b) to be true iff $p(x) > .5$ where x is *If the car crashes at greater than 35mph, the airbag will go off*. It seems that as a matter of fact (5-b) is judged true iff the conditional probability that the airbag going off given that the car crashes at greater than 35mph is high. Recall that where $p(y) \neq 0$ the conditional probability of x given y , $p(x|y) = \frac{p(x \wedge y)}{p(y)}$. To see that the judgment in (5-b) depends on conditional probabilities think about two sorts of case. In the first, you think it's very likely

⁴For a recent critical assessment see Yalcin (2007). I think what one can conclude from the recent literature is that the propositionalism about epistemic modals, to be viable, needs to posit a lot of rather non-standard context-sensitivity in the choice of attitudes.

⁵This issues relate closely to the question in the previous section of what sort of semantics we should give to epistemic modals.

that the car will crash at greater than 35mph, but you think it's very unlikely that both the car will crash at greater than 35mph and the airbag will go off. Then your conditional probability of the airbag going off given that the car crashes at 35mph cannot be very high, and correspondingly you must judge (5-b) as false. By contrast, if you think it's likely the car will crash and only slightly less likely that the car will crash *and* the airbag will go off then you would need to judge (5-b) as true. Generalizing away from this example, it's intuitive that *it's likely that if a then c* is true iff the conditional probability of *c* given *a* is greater than one half. Our prior assumptions about the meaning of sentences of the form *it's likely that x* along with this last observation, should lead us to the conclusion that conditionals express propositions whose probability is just the conditional probability of the consequent given the antecedent. This hypothesis is what is often called *the equation* or *Adams's thesis*: the probability of "if *a* then *c*" is equal to $p(c|a)$ (see Edgington, 1995, and references therein). The problem with this conclusion noted first by Lewis (1976), is that, in a certain sense, *there is no proposition that satisfies Adams' Thesis*.

Here's the sense:⁶ suppose we assume that there's a set of possible worlds W , and any given sentence, x , expresses a proposition by picking out a subset of W , i.e. the worlds where x is true. Let's then say that any given credal state is a probability function defined over the power-set (or some sigma algebra) of W , i.e. a function that tells you for any given set of possible worlds how much credence you have that the actual world is one of those. Now let $a \rightarrow c$ be *if the car crashes at greater than than 35mph, the airbag will go off*, and a and c be the antecedent and consequent respectively. Since $a \rightarrow c$, a , c are all sentences that we can have credences in, they each pick out a subset of W . Suppose that on someone's credal state, p , $p((a \rightarrow c) \wedge \neg a) > 0$, and $p(c|a) \neq p(c) \neq p(a)$, and none of those values are 0 or 1. Further suppose, as suggested, that $p(a \rightarrow c) = p(c|a)$. Now, it is easy to show that there will exist another probability function p_1 such that $p_1(a \rightarrow c) \neq p_1(c|a)$.⁷

What's the problem with this? Well, what we've just shown is there isn't any subset of W to which every probability function assigns a probability equal to the conditional probability of c given a . This suggests that our semantic theory will not be able to assign a *general* meaning to $a \rightarrow c$ which 1) applies across different credal states *and* 2) fits into the natural account of the semantics for sentences of the form *it's likely that x*. This is not a happy situation, since $a \rightarrow c$, intuitively, has some sort of uniform meaning.⁸

⁶This is a variation on Lewis's first "triviality theorem" (Lewis, 1976). There are actually a variety of related arguments that go from Adams' thesis and some auxiliary assumptions to some form of contradiction (Hajek and Hall, 1994).

⁷One can construct p_1 , for instance, by making it the result of conditionalizing p on $\neg(\neg a \wedge (a \rightarrow c))$. It will then follow that $p_1(a \rightarrow c) < p(c|a)$, since $p_1(a \rightarrow c) < p(a \rightarrow c)$ but $p_1(c|a) = p(c|a)$.

⁸Some have argued that the proposition expressed by a conditional sentence varies with the epistemic state of the speaker. In this case, the argument I gave in the previous paragraph would have no force. There are, in fact, further problems with maintaining Adams' thesis even if one allows conditionals to express different propositions relative to different credal states, but I won't discuss them here. See Edgington (1995) and Bennett (2003) for discussion and citation of the major results.

If no proposition that a conditional could express has the right probability, then we have a powerful argument that conditionals do not, in fact, express propositions. If that's right then the relationship between probability and conditionals yields a formidable consideration in favor of the attitude view of semantics, at least with regard to conditionals. Certainly this is the position taken by Adams (1975), Edgington (1995), and Bennett (2003). These earlier formulations of attitude views did not assign systematic semantics to conditionals, but more recent work such as Yalcin (2007) and Swanson (2006) has given more compositional treatments of conditionals along these lines.

Before getting to the main proposal of this paper, a propositional semantic for conditionals which captures Adams' thesis, I will consider two related ways of escaping Lewis's argument for those who want to assign truth-conditions to conditionals.

2.1 Syntactic Account

Kratzer (1981, 1986) denies the syntactic parsing that is required to formulate this problem.⁹ She claims that the function of the 'if'-clauses is to restrict higher-up modal quantifiers in the sentence. In the case of (5-b) the natural choice of the operator to be restricted by the 'if'-clause is the probability operator *it's likely that*. Thus, when we parse (5-b) we do not, in fact, treat the conditional as one unit, but, rather, we treat the antecedent, 'if the car crashes at greater than 35 mph,' as a restrictor on the probability operator and treat the consequent, 'the airbag will go off,' as what the probability operator is applying to. If we make normal assumptions about the semantics of probability operators, then this should give us a good compositional semantic treatment of (5-b).¹⁰

Kratzer's account is very attractive. There is good evidence that certain sorts of constructions are naturally handled by thinking of antecedents as capable of restricting certain higher-up operators. At the least, this seems to be how one should understand the interaction of conditionals and adverbs of quantification in such examples as the following, which motivated Lewis's original account:

- (6) Usually/always/sometimes, if/when there is an accident, the police come within five minutes.

Kratzer goes beyond Lewis, and proposes that *unembedded* indicative conditionals are also of this form, except that the modal in these cases is a silent epistemic necessity operator. Consider the pair of an indicative conditional and a probability judgment about it:

- (7) a. If the car crashes at greater than 35mph, then the airbag will go off.

⁹Her work is an extension of the treatment of 'if'-clauses in sentences with adverbs of quantification in Lewis (1975).

¹⁰Cozic and Égré (2009) is a detailed discussion of the relation between Adams' thesis and Kratzer's semantics.

- b. It's likely that if the car crashes at greater than 35mph, then the airbag will go off.

On Kratzer's view the probability judgment in (7-b) is not being made about (7-a) directly since the presence of the overt probability modality operator means we do not need the silent epistemic modal! The two logical forms are as follows:

- (8) a. \Box_{epist} [car crashes][airbag goes off]
b. LIKELY [car crashes][airbag goes off]

This has the odd consequence that when judging the truth of (7-b) we are not evaluating the probability of (7-a). This is, of course, the trick needed to escape Lewis's triviality results: on this account there is no proposition whose probability satisfies Adams' thesis. But this does leave us with the somewhat odd result that the appearance of similarity in form between the pairs (5-a) and (5-b) is in fact illusory. Her theory thus has the unintuitive consequence that while conditionals express propositions, when judging the probability of a conditional we do not need to think about the probability of the proposition that the conditional expresses.

When we consider the fact that judgments of probabilities go beyond judging the truth-conditions of sentences with overt probability operators, the oddness of the view becomes more apparent. Consider again (7-a). On Kratzer's view this expresses a proposition, one which is true in some worlds and false in others. We can, it would seem, *think* (without necessarily accessing any particular sentence) about the probability of this proposition. Intuitively, in this case, the probability of (7-a) is just the conditional probability, in accordance with Adams' thesis. But for Kratzer's view to explain Adams' thesis she needs to maintain that when we evaluate the probability of (7-a) we do *not* think about how likely it is that it's true. Instead, somehow, whenever we make a probabilistic judgment of a conditional there is an operation of syntactic restriction whereby the antecedent of the conditional restricts the probability modal. It is not easy to understand exactly what this amounts to in the case of actual judgments in *thought*, which we do not always think of as having syntactic modals expressing probabilities. So, even if Kratzer is right that antecedents of conditionals often serve as restrictors for modal operators, it is not at all clear how this story extends to explain our judgments of the probabilities of conditionals.

3 Antecedents as Context Shifters

If we allow ourselves some freedom with the syntax of judgments of probabilities of conditionals, other options emerge for explaining Adams' thesis without falling prey to Lewis's triviality results. One way to capture Adams' thesis without adopting Kratzer's view of the syntax of conditionals is to think of probability operators as always getting embedded inside the consequent of the conditional. So when we judge that the probability of $a \rightarrow c$ is high,

what we are really saying is that we deem $a \rightarrow$ (the probability of c is high) to be true.

If we allow this understanding of what probability judgments of conditionals amount to, then in order capture Adams' thesis we simply need to get the semantics of conditionals to yield the result that we only judge $a \rightarrow$ (the probability of c is high) to be true when our conditional probability of a given c is high. There are a variety of options for both propositional and non-propositional kinds of semantics that will yield this result.¹¹ One such propositional view has it that $a \rightarrow c$ is true just in case c is true in the context achieved by adding a . So, in other words, $a \rightarrow$ (the probability of c is high) is true just in case when you alter the context by adding a , the probability of c is high. The reason this semantics will capture Adams' thesis is just that what it means to have a high conditional probability of a given c is that were one to assume a then one's conditional probability of c would be high. Of course, much needs to be said about the details of the contexts and context-shifts to spell out this view, but perhaps this is enough to give an idea of how it works.¹²

If this style of view is going to explain Adams' thesis, like Kratzer's view, it must resort to saying that when we judge the probability of a conditional we are not actually judging the probability of the proposition the conditional expresses.

The view I present below differs from both Kratzer's view and the context-shifting views in that it does not give a revisionary account of what it is to judge the probability of a conditional: on the account I propose it is just to assess the probability of the proposition the conditional expresses.

4 A Classical Semantics

Here, I propose another way of explaining the relationship between conditionals and probabilities. This method takes a standard semantics for indicative conditionals, essentially that of Kratzer (1986) but without making use of the special syntax of antecedents. On this semantics, indicative conditionals are context sensitive, but within a given context they express a proposition which has a probability that we can make judgments about in the usual way. I then show how certain assumptions about the truth-conditions of indicative conditionals can, in many cases, validate the judgement that the probability of a conditional is its conditional probability. I defend these assumptions and show where we must depart from

¹¹One of each is reviewed in Gillies (2009): he does not specifically discuss Adams' thesis but the semantics he gives will capture Adams' thesis if one makes standard assumptions about the semantics of probability operators. Yalcin (2007) also gives a version of this kind of view and shows that it predicts the links between conditionals and conditional probabilities.

¹²In particular, if we don't want antecedents to actually shift the actual context, we need to use double-indexing (Lewis, 1980). We also need to make sure the context itself contains enough information to give truth-conditions to probability statements. All this is non-trivial work, but I don't think there are any problems in principle here. Yalcin (2007) spells out most of the needed assumptions explicitly, and while the view he gives is non-propositional he explains how it can be altered into a view on which conditionals and probability operators express propositions. Klinedinst and Rothschild (2010) expand on Yalcin's semantics to cover context shifts affected by other connectives besides conditionals.

the assumptions behind Lewis's triviality proofs in order to avoid contradiction. Needless to say, this approach has its own problems, but even if it is not ultimately the correct semantics for conditionals it is important to see how certain propositions naturally have as their probability the conditional probability of two related propositions.

4.1 Strict Conditionals

A number of semantics for indicative conditionals, *strict conditional* analyses, treat conditionals as restricted necessity modals, i.e. as expressing the necessity of the consequent in all cases in which the antecedent is true (e.g. Kratzer, 1986).¹³ To see one reason why this idea is attractive, consider these two sentences:

- (9) a. John must be here.
b. If Mary is here, then John is here.

A natural thought is that (9-a) expresses the epistemic necessity that John is here and (9-b) expresses the restricted epistemic necessity: in all epistemically possible worlds where Mary is here, John is here. According to Kratzer (1986) there is a hidden epistemic necessity modal in all unembedded indicative conditionals and the antecedent acts as the restrictor of the modal and the consequent as its matrix clause. What remains is to give a semantics for epistemic necessity modals.

For the reasons sketched in section 1 we will treat epistemic necessity modals as statements about knowledge states. We will call the particular the source of knowledge in question X and formulate our semantics for conditionals and epistemic modals as claims about X :

Classical Semantics for Epistemic Modals and Indicative Conditionals

- $\Box p$ is true iff p is true in every world w compatible with X 's knowledge (more simply put: ' X knows that p ')
- $p \rightarrow q$ is true iff $p \supset q$ is true in every world w compatible with X 's knowledge (more simply put: ' X knows that $p \supset q$ ')

The status of X is controversial. Kratzer (1986) proposed that (for conditionals) X should be identified with the speaker, but there are numerous examples that this proposal cannot handle.¹⁴ I will posit a much more abstract identity for X . X is a context-sensitive source of knowledge, the extent of which depends on the conversational participants but often goes

¹³Note that even some non-classical approaches to indicative conditionals and epistemic modals draw this same connection. See particularly Yalcin (2007) and Gillies (2004).

¹⁴See, for instance, the discussion in von Stechow and Gillies (forthcoming), where practically every possible value is tried and rejected and the paper concludes by maintaining the classical approach to epistemic modals only at the cost of largely rewriting the rules of assertion for these constructions. See also Yalcin (2007) for a presentation of a particularly thorny problem for any semantics of epistemic modals along these lines.

beyond it. Hence, there is some sense in which X represents an idealized, but still limited, source of knowledge. One way of thinking about this source is to take $a \rightarrow c$ to mean “it is known that all a -worlds are c -worlds” which perhaps better captures the “objective” flavor of the knowledge source than the use of X does.¹⁵ Below I will propose some more concrete principles governing the choice of X that explain the link between conditionals and conditional probability.

4.2 The Nature of X and the Probability of Conditionals

As mentioned, I will not use Kratzer’s *syntax* for conditionals embedded under probability operators. Rather, we will assume that conditionals have the truth conditions above and try to figure out what probability we should assign to them, just as we would assign probabilities to any other proposition. Even if you think Kratzer’s syntactic approach is right in many cases, this project might still be of interest, since you might wonder what probability we should assign to Kratzer’s unembedded indicative conditionals.

Let me first note something obvious: if we allow X to be the speaker and we think of probabilities as being subjective probabilities for the speaker then the probability of conditionals will often be either zero or one. For example, consider the conditional (7-a). If X is just me, the speaker, then (7-a) is true just in case I know that any case in which the car crashes at greater than 35mph will also be a case in which the airbag goes off. My judgment about the probability that I know something will (in the normal case, putting aside cases in which one does not know about one’s own beliefs) be either 0 or 1. So we will be hard-pressed to find a case where we can say that (7-a) is likely but not certain. In this case, we cannot preserve the observed connection between conditionals and conditional probability.

So X will have to *not* be the speaker if we are to find non-trivial probabilities for indicative conditionals on this semantics. As I said, we will instead think of X as a sort of idealized (but not omniscient) knowledge source.¹⁶ Obviously there will also be some context sensitivity about the extent and nature of X ’s knowledge since epistemic modals are context sensitive: so the knowledge source X must reflect, in some way, the speaker and hearer’s knowledge even if it does not correspond exactly to either one (or the combination of the two). I have no particular theory about the nature of X , but I will make a few posits which will do the necessary work:

Properties of the Knowledge Source for Indicative Conditionals Whenever one utters an indicative conditional $a \rightarrow c$ the following default assumptions about X are made (when probabilities are mentioned they are with respect to the speaker’s subjective probabilities):

¹⁵Thanks to Brett Sherman for suggesting this formulation.

¹⁶This idea, I suppose, might be floating around somewhere in the literature.

1. If a is true then all the a -worlds compatible with X 's knowledge are either all c -worlds or are all $\neg c$ -worlds. On our semantics for conditionals this amounts to what is often called strong centering: $a \supset (c \leftrightarrow (a \rightarrow c))$
2. The probability that all a -worlds compatible with X 's knowledge are c -worlds is independent of the probability that a is true.¹⁷ This amounts to the independence of the antecedent from the consequent: $p(a \rightarrow c)p(a) = p((a \rightarrow c) \& a)$

To better explain and justify these assumptions I will illustrate them with an example of a *real world* idealized knowledge source. Consider this conditional said of a certain car:

(10) If the car crashes at greater than 35mph, the airbag will go off.

Imagine that there is a defect for cars of this sort such that cars with the defect have airbags that wouldn't go off at 35mph crashes but cars without the defect have airbags that would go off in such crashes. Imagine that our idealized source of knowledge, X , knows whether or not the car has this defect and knows the effect of the defect on crashes, but that he doesn't know whether or not the car will crash in the future. Suppose, moreover, that the conversational participants don't know whether or not the car has this defect. This seems like a pretty typical case of an idealized source of knowledge: the knowledge source knows a bit more than us about the workings of the car but he is not omniscient. As before, let $a \rightarrow c$ mean that according to this X all a -worlds are c -worlds. Given our example it is reasonable to make both assumption 1 and 2 above about X . Assumption 1 simply follows from the descriptions, and assumption 2 is reasonable since we have no reason to think that X 's beliefs about the car having the defect depend in any way on whether or not the car will have an accident, so we should assume the probabilistic independence of these two questions. (To assume they were *not* independent would amount to thinking that X would be more (or less) likely to know about defect if the car were going to crash, and this would be a strange thing to think without further information.)

It follows from assumptions 1. and 2. that $p(a \rightarrow c) = p(a|c)$.¹⁸ This may seem intuitively obvious, but it's worth going through the proof:

Proof. Assumption 1. can be expressed as follows:

$$a \supset (c \leftrightarrow (a \rightarrow c)) \tag{a}$$

The assumption of probabilistic independence of the knowledge source from the antecedent (assumption 2.) can be expressed as follows:

$$p((a \rightarrow c) \wedge a) = p(a)p(a \rightarrow c) \tag{b}$$

¹⁷By definition, the probability of a is independent of the probability of c iff $p(a \wedge c) = p(a)p(c)$.

¹⁸This observation is closely related to that of Ellis (1978) who shows that the Stalnaker conditional satisfies Adams' thesis when the probability of the antecedent is independent of the probability of the conditional itself.

If we divide each side of (b) by $p(a)$ we get:

$$\frac{p((a \rightarrow c) \wedge a)}{p(a)} = p(a \rightarrow c)^{19} \quad (c)$$

Because of (a) we can substitute $p((a \rightarrow c) \wedge a)$ in c with $p(a \wedge c)$ giving us the needed equivalence:

$$\frac{p(a \wedge c)}{p(a)} = p(a \rightarrow c) \quad (d)$$

Given the standard definition of conditional probability we get the equation (Adams' thesis).

$$p(a \rightarrow c) = p(c|a) \quad (e)$$

□

What this demonstrates is that for certain choices of knowledge source X it is necessary to assign the same probability to the proposition that the knowledge source X thinks all a -worlds are c -worlds as one assigns to the conditional probability that a given c .

4.3 Supporting the Assumptions

I will give some more motivation for assumptions 1. and 2. above. Actually these assumptions have rather different statuses: The probabilistic independence of the antecedent from the conditional itself is quite plausible on its own as a default assumption for our semantics. However, I will argue that independence does not always hold, and that this is the reason we do not have a contradictory system. Strong centering, on the other hand, seems to be built in to our semantics for conditionals.

4.3.1 Probabilistic Independence

I think more can be said for these assumptions beyond the work they do. The probabilistic independence assumption is the most innocuous. It simply states that $p(\text{every } a\text{-world compatible with } X\text{'s knowledge state is also a } c\text{-world and } a \text{ is true})$ is equal to $p(a \text{ is true})$ times $p(\text{every } a\text{-world compatible with } X\text{'s knowledge state is also a } c\text{-world})$. This is reasonable when we don't think X 's knowledge state is causally linked or contextually entailed by the question of whether the antecedent is true, which it is reasonable to assume in our example. The point is, if there is no such causal link or contextual entailment then our learning the antecedent is true or false should have no effect on our confidence in the propositions about

¹⁹Note that here I am assuming that $p(a) > 0$. I assume, like many others, that the felicity conditions for asserting an indicative conditional involve (at least for the purposes of conversation) assuming that the antecedent is possible.

X 's knowledge.²⁰ As we'll see, however, it is crucial to the consistency of our account, as well as its predictive power, that this assumption sometimes fails.

4.3.2 Strong Centering

Strong centering is widely accepted in the literature as a principle governing the logic of conditionals. It simply makes the truth value of the conditional equal to the truth of the consequent in worlds where the antecedent is true. The only case in which this might seem dubious is when the consequent is, as it were, true by accident, and even here it's not clear that we would judge the conditional to be false.²¹

There is also good empirical evidence that strong centering holds in our semantics. For, on this semantics, strong centering is entailed by the *conditional excluded middle* (CEM):²²

$$(a \rightarrow c) \vee (a \rightarrow \neg c) \quad (\text{f})$$

There is good evidence that CEM is a semantic rule that we use for reasoning about conditionals (von Fintel, 1997; von Fintel and Iatridou, 2002). The most basic observation supporting this view is that negated conditionals seem to behave like conditionals with the same antecedent but the negated conclusion. Consider this sentence for example:

(11) I doubt that if John takes the exam he'll pass.

It seems that (11) entails that I believe that if John takes the exam he'll fail. The best explanation of why this is so is that the conditional excluded middle holds: so if "If John takes the exam then he'll pass" is false then "If John takes the exam, then he won't pass" is true, and hence not believing one amounts to believing the other. This seems like good evidence that the CEM is taken for granted by speakers.

I should note that strong centering is slightly stronger than what we really we need for our proof. We could just use this instead:²³

$$p(a \wedge c) = p(a \wedge (a \rightarrow c)) \quad (\text{g})$$

This could be true even if strong centering is false, so, e.g. in cases where you allow a and c to be true but $a \rightarrow c$ to be false. I am not sure, though, that there is any independent motivation for the probabilistic version of strong centering.

²⁰I'm not saying causal links or contextual entailments are the only reasons to not have probabilistic independence, but that they would be the obvious ones in this kind of situation. Probabilistic independence is not reducible to something else, but it can be a default assumption when two things are causally independent.

²¹These cases are discussed by Lewis (1973).

²²CEM entails strong centering because on our semantics a and $a \rightarrow c$ together entail c , given the factivity of knowledge.

²³This is what Ellis (1978) uses for his proof.

4.4 Idealized Knowledge Sources

So both assumptions have something supporting them. Nonetheless you might wonder why we should think that indicative conditionals and epistemic modals make reference to context-sensitive idealized knowledge sources at all.

One piece of evidence is that epistemic modals seem to involve knowledge (or inferences, at least!) that go beyond any particular conversational member's knowledge (or the combination of all of their knowledge). Suppose for instance, we had mistakenly calculated, based on some ships logs, that the wreck could have happened at some spot. We might then utter:

(12) The wreck might have happened here.

But there is an obvious sense in which what we say is simply false, since it is based on a miscalculation, thus there seems to be an appeal to knowledge sources that go beyond our own.²⁴

Let me also briefly explain how positing idealized sources of knowledge can yield prima facie plausible conditions for the felicitous assertion of epistemic necessity modals and conditionals. Obviously, the conditions for *asserting* an epistemic modal or indicative conditional include *knowledge/belief* that it is true. In the case of epistemic necessity modals, this will require knowing the truth of the prejacent, and, in the case of conditionals, requires knowing that at least the material conditional is true. If we assume that the idealized knowledge sources always have more knowledge than the speaker, then these will be not only necessary but also sufficient conditions for assertion for these two constructions. The theory that posits an idealized knowledge source, thus, makes plausible enough predictions for the conditions of assertions of epistemic necessity modals and conditionals.²⁵

So we have seen that using idealized knowledge sources in our semantics for epistemic modals and conditionals both explains some of the 'objective' feel of modals and yields plausible enough conditions for assertion.

The main rival to this sort of semantics are what I labelled the attitude view of semantics, which views epistemic modals and conditionals as expressing states of mind or urging some sort of change to the common ground, such as the accounts of Veltman (1996), Swanson (2006) and Yalcin (2007).²⁶ These non-classical theories, while very appealing in many respects, have their own troubles. In particular, they must struggle to explain why and how epistemic expressions can be embedded in contexts that take items of propositional values. The theories I mentioned all give compositional semantics for complex expressions involving conditionals or models, so the problem is not one of compositionality itself. Rather it is

²⁴See von Stechow and Gillies (forthcoming) for more on this.

²⁵Modulo the issues about when one cannot assert a conditional because the antecedent is known to be false, but this is a common problem for many semantics of conditionals.

²⁶I lump dynamic theories with the attitude views for reasons discussed in Yalcin (2007).

to explain what it means to use these expressions embedded in various contexts, such as in questions:

- (13) a. Will John come if Susan does?
 b. Must John be on the boat?

These questions are perfectly sensible if we understand them as asking about what sources of knowledge better than us know. Of course, the attitude view (or its variants), which I am sympathetic to, may be augmented to explain such uses of epistemic modals, but what I am pointing out here is that it is not as natural for them to do so as it is for the classical account that I have outlined.

5 Disrupting The Triviality Proofs

Of course, if we allow Adams' thesis to be true across the board, by Lewis's argument, we will turn out to have inconsistent probabilistic beliefs. So if our two assumptions, strong centering and independence, entail Adams' thesis, then these assumptions cannot always hold, or we will have a contradiction. I will try to argue that in the very cases where these assumptions might lead us to contradiction, we can see that the independence assumption is independently implausible. Thus, we have an explanation of why, despite these assumptions holding as *defaults*, they do not in fact lead us into contradiction.

To simplify, I'll reconstruct a version of Lewis's simplest demonstration of an absurdity that follows from Adams' thesis. We'll see that the proof uses a slight generalization of Adams' thesis, and this generalization is what we need to challenge. Here is one version of the proof.²⁷

$$p(a \rightarrow c) = p(a \rightarrow c|c)p(c) + p(a \rightarrow c|\neg c)p(\neg c) \quad (\text{h})$$

$$p(a \rightarrow c) = p(c|a \wedge c)p(c) + p(c|a \wedge \neg c)p(\neg c) \quad (\text{i})$$

$$p(a \rightarrow c) = 1 * p(c) + 0 * p(\neg c) \quad (\text{j})$$

$$p(a \rightarrow c) = p(c) \quad (\text{k})$$

$$p(c|a) = p(c) \quad (\text{l})$$

The conclusion is obviously an absurdity since we have proved with no further assump-

²⁷I've taken this exact formulation of Lewis's proof from Paul Égré.

tion that the conditional probability of c given a is the probability of c ! On our classical account of the conditional connective no steps may be plausibly challenged except the move from (h) to (i) which depends on the *exportation* principle that $p(a \rightarrow c|b) = p(a|b \wedge c)$. Of course, this principle just seems like a generalization of Adams' thesis, and a plausible one at that. In fact, Lewis (1976) justifies this principle by simply applying Adams' Thesis to the new probability function found by conditionalizing on b .²⁸ If we are to object to this thesis, then we need to explain why such a move is not acceptable in general.

One reason, of course, to object to the exportation principle, is that it leads to absurd consequences, as Lewis showed. But it would be nicer to have a more principled understanding of why we should reject it on our semantics. To see why it is worth going back to our toy example, and trying to make sense of Lewis's proof in this case. Recall the notation: a = car crashes, c = airbag goes off, and $a \rightarrow c = X$ knows that all a -worlds are c -worlds. Obviously factorization is acceptable so we can state this formula with confidence.

$$p(a \rightarrow c) = p(a \rightarrow c|c)p(c) + p(a \rightarrow c|\neg c)p(\neg c) \quad (\text{m})$$

What is the probability that X knows that $a \supset c$ given $\neg c$? According to Lewis, using his exportation principle, it is 0. However, this is clearly not the case: even if the airbag does not go off, we may still think that X knew it would have gone off if the car had crashed (since X may have known the car had the defect). So it is clear in this case that $p(a \rightarrow c|\neg c) \neq p(c|\neg a \wedge a)$. What this amounts to, then, is a denial of Adams' thesis applying for this choice of X in the probability function reached by conditionalizing on $\neg c$. Why does Adams' thesis not hold for this new probability function? Suppose we learn the airbag will not go off (i.e. $\neg c$). Can we still maintain the assumption that the question of whether X knows $a \supset c$ is probabilistically independent of a ? Obviously not: if a is true than X must *not* know $a \supset c$ on pain of contradiction.

So our previous derivation of Adams' thesis does not work since it depended on the crucial independence assumption which is disrupted by our learning that $\neg c$. For the new probability function p' (arrived at by conditionalizing on $\neg c$), it is not true that $p'(a \rightarrow c) = p'(c|a)$, for the latter is simply 0 while the former is positive. Thus, we cannot use the background assumptions that supported the simple (unconditionalized) case of Adams' thesis to also recreate Lewis's triviality proof. I take it this is a welcome result for the classical approach, though it highlights the extent to which, on the classical approach, we cannot make Adams' thesis a semantic rule.²⁹

Of course, it has long been known that if we somehow restrict the application of Adams' thesis we can avoid the contradictions from the triviality theorems. The contribution of

²⁸Note that this is exactly what we did to derive the contradiction in footnote 7.

²⁹I must reserve discussion of other triviality results, such as those of discussed in Hajek and Hall (1994) for another place. My review indicates that none of these results undermines the proposal here, but this is just a preliminary sense.

this explicit semantics with its particular understanding of independence is to explain *why* Adams' thesis should only hold some of the time.

6 When Independence Fails...

I will now argue that our account not only captures Adams' thesis in many cases but also has the potential to explain the cases in which Adams' thesis fails. This makes the empirical coverage of our account stronger than that of other accounts, such as that of Edgington (1995), which essentially hard-wire in Adams' thesis.³⁰

The assumption that the antecedent of the conditional is probabilistically independent from the conditional itself (i.e. the knowledge claim made by the conditional) is absolutely crucial to this account. But what about cases where such an independence would seem to fail?

Take our original car defect example. Suppose that the defect not only makes it so that the airbag does not go off, but that it also makes it the case that the car is more likely to crash. In this case, the fact that the car crashes increases one's credence in the proposition that the car has the defect (and hence that X knows that all crash-worlds are airbag-not-working-worlds).

In this case, then, then $p(c|a)$ will be greater than $p(a \rightarrow c)$, so Adams' thesis fails. But oddly enough, this judgment seems like it is supported by intuition. It seems like the probability that if *this* car crashes its airbag will go off just is the probability that it lacks the defect (at least on one salient reading of the conditional). The problem is that were you to learn that the car had crashed you would increase your credence in the proposition that the car had the defect. But in our initial position it seems that we assign a higher probability to the proposition that if the car crashes its airbag will go off, than we would actually assign to the proposition that the car's airbag will go off in the case in which we learn that the car has been in an accident. So, in this case, Adams' thesis fails, which is actually what our theory predicts.³¹ As Kaufmann (2004) notes, it seems like conditionals in these sorts of situations are ambiguous between readings that support Adams' thesis and readings that don't. We might be able to model that ambiguity by an ambiguity in the relevant X : we can consider an objective knower that is independent of the antecedent, but we can also conceive one that is not. Obviously this corner of the world of conditionals and probability requires more systematic exploration, but the examples considered suggests that, as the classical account here predicts, Adams' thesis may fail when independence fails.

³⁰These types of cases are discussed extensively in Kaufmann (2004). That such potential counterexamples to Adams' thesis exist was pointed out to me in conversation by John Hawthorne, who independently developed his own examples.

³¹I'm slightly tempted to think this is an instance of cognitive error of some sort, akin to base-rate neglect. Obviously if this is so, then these cases don't support the view of conditionals put forward here.

7 Interactions between Conditionals and Modals

Consider this set of examples:³²

- (14) a. If the coin was flipped it came up heads.
b. If the coin was flipped, it must have come up heads.
c. If the coin was flipped, there's a 50% chance it landed heads.

If the coin is fair and we have no further information it is natural to judge (14-a) as having a 50/50 chance of being true, (14-b) to be false, and (14-c) to be true. Kratzer, of course, has a simple account of all of these facts: in (14-b) and in (14-c) the modal operator is restricted by the antecedent of the conditional and our probability judgment about (14-a) is actually an instance where the restrictor of (5-b) restricts the probability judgment we make (as discussed in Section 2.1). The one problem her account faces is that we can, it seems, discard the silent epistemic modal when we evaluate the probability of (14-a), but we cannot discard the explicit modal when we evaluate the probability of (14-b).

What can we say on the classical account, though, about the pattern of probabilistic judgments of the sentences in (14)? One strategy is to adopt Kratzer's syntactic analysis of (14-b) and (14-c), thus giving us the requisite readings for those two examples. We are only left with the problem, which Kratzer shares, of explaining why (14-a) and (14-b) don't have the same probabilities. The answer might be that the explicit modal "must" has a different semantics from the silent epistemic modal in (14-a). This is actually not that implausible: it is often observed that epistemic "must" seems appropriate only when what is embedded under it is known by indirect evidence rather than direct evidence. For example, if you can see that it is raining, it is a bit strange to say "It must be raining." Since no similar phenomenon seems to apply to indicative conditionals, it is natural to think the silent modal we posited in indicative conditionals differs from the epistemic "must".

Another strategy is to take the surface syntax of *all* these examples at face value. In this case, (14-b) and (14-c) both are instances of conditionals whose consequents contain epistemic modals. So, in these cases we would then need to posit two different knowledge sources: that for the conditional itself and that for the modal in the consequent. We can write out the logical forms then as follows, where X is the knowledge source of the indicative conditional and Y is the knowledge source of the embedded modals in the consequent:³³

- (15) a. $\Box_X (\text{coin flipped} \supset \text{heads})$
b. $\Box_X (\text{coin flipped} \supset \Box_Y \text{heads})$
c. $\Box_X (\text{coin flipped} \supset p_Y(\text{heads}) > .5)$

³²I am grateful to Benjamin Spector for pointing out the potential issues here, particularly with regard to (14-b), below.

³³More notation, in case it isn't clear: \Box_A means "A knows that" and p_A is the probability function according to A.

But if these are the meanings, why do we have the various different judgments about their probabilities? To get clearer about this, it is useful just to think of the contrast between the three consequents in cases where we know the antecedent is true. Consider these three sentences said about an event in which a fair coin was flipped:

- (16) a. It landed heads.
b. It must have landed heads.
c. There's a 50 % chance it landed heads.

I take it that with no further information, we would naturally rate (14-a) as having a 50/50 chance of being true, (16-b) as being false, and (16-c) as being true. What happens then if, instead of taking for granted that the coin is flipped, we embed these sentence in a conditional construction, with the semantics sketched above? Recall that our idealized knowledge source is posited to be an extension of our own knowledge. So it is not surprising that in the cases we are certain of, namely, (16-b) and (16-c), the judgment we get is the same as the one we get in the unembedded case: namely that all coin flipping worlds are ones in which (16-b) is false and (16-c) is true.

Of course, this is slightly subtle: the knowledge source inside the consequent in (14-b) and (14-c) cannot be simply our knowledge from the context of uttering those sentences, but rather it must be what our knowledge would be were we to learn that the antecedent is true. So we need to make the knowledge sources for the modals inside the conditional extensions of ours with the added knowledge that the antecedent is true. The idea that the actual sources of knowledge used by modals shift inside embeddings may seem like an outrageous posit. However, it turns out we will need to say something like this for constructions that don't involve conditionals at all, so the idea is independently motivated. Consider, for instance, this disjunction with an epistemic modal in the second disjunct.

- (17) Either John is in the basement, or he must be in the kitchen.

If we read the "must" in (17) as being about the same knowledge source the knowledge source "must" would pick out in an unembedded case, we would get a bad reading for (17).³⁴ The argument for this is simple: we could judge (17) to be certainly true but nonetheless judge the two embedded disjuncts of (17) to be respectively uncertain and certainly false. This set of judgments, however, is impossible on a normal semantics for disjunction. So either epistemic modals need to shift their knowledge bases in some embeddings or we must reject the normal semantics of disjunction. I'm actually not sure which option is right, but I take it that it is, at least, plausible that epistemic modals (and probability modals) shift

³⁴It seems to me that the information that can be added to the knowledge base in these contexts is related to what information is in the local context in the sense of local context used in the study of presupposition projection, e.g. Heim (1983); Schlenker (2009). Klinedinst and Rothschild (2010) expand on this idea to give a compositional static treatment of (17).

their knowledge bases when embedded in conditionals and disjunctions.³⁵

What about with (14-a)? In order for the conditional to have a probability of 50% we need to assume that the knowledge source X knows (in half the possible worlds) that any coin-flipping world is a heads world. This is fine as long as X 's knowledge extends beyond our own.

Now we might wonder whether there is a single knower X that could be operative in all three conditionals. That depends on the meaning of the embedded modals. If they all refer to our own potential knowledge states in the way I suggested above, then there is no problem. It would be good to have clearer principles about how we choose the knowledge sources for different modal and conditional constructions. It is certainly not the case that explicit modals are the only ones that go beyond our own knowledge. As I've suggested, some uses of "must" also seem to refer to knowledge sources that extend beyond our own. But the knowledge source of an indicative conditional can extend *even further*. As we saw, to get the probability of (14-a) right we need to imagine a knowledge source which actually (half the time) knows how the coin would have landed if it had been flipped!³⁶ My tentative hypothesis is that only the silent epistemic modal of conditionals can include knowledge that extends this far; explicit modals like "must" or "it's likely that" never allow knowledge of this kind.

It's perhaps worth pointing out that we can now see why, whether we place the modal "it's likely" inside or outside of an indicative conditional, we get no difference in truth conditions.

- (18) a. It's likely that if the coin was flipped it landed heads.
b. If the coin was flipped, it's likely it landed heads.

Assume in both cases that the knowledge source of "it's likely" is just our own knowledge (or what we would believe if we learned the coin was flipped). On the other hand, the knowledge source X of the conditional we will assume is such that it actually is hypothesized to know in all cases how the coin landed (or would have landed) were it flipped. In this case both (18-a) and (18-b) will be true in the same circumstances: those in which we believe that the coin is likely to land heads when flipped.³⁷

³⁵As a note, this shift in the modal basis is also the centerpiece of a recent strict-conditional account of conditional given by Gillies (2009). He shows how these shifts can allow strict conditional accounts to handle right-nested conditionals such as $a \rightarrow (b \rightarrow c)$.

³⁶This knowledge might even turn out to be physically impossible in cases in which there is real indeterminacy. I take it that we shouldn't be too bothered by the idea that we sometimes posit physically impossible knowledge.

³⁷To show this for (18-a) we just need to follow the reasoning presented in this paper to capture Adams' thesis. For (18-b), just note that it is true on the semantics proposed if and only if an idealized knowledge source X knows that were we to learn the coin was flipped we would find it likely to land heads. Assuming the knowledge source accurately knows our beliefs then this would seem to be true just in case, were it to be flipped we would judge it likely to land heads.

8 Conclusion

The problem with conditionals and probability should not be seen as an insurmountable one for the propositional view of indicative conditionals. We saw that a standard semantics for conditionals with a few well-supported assumptions seemed to be able to explain Adams' thesis in a way that might not cause any worries about triviality. Moreover it turned out that, in certain cases, the theory predicts that Adams's thesis fails, and this prediction actually seems like it might just be right.

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