

Quantified Epistemic Modality

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Explaining Contradictory Sentences

Let's begin with Moorean contradictions:

- (1) a. It's raining but I don't know it's raining.
- b. It's raining but I don't believe it's raining.
- c. It's raining but it might not be raining.

These have familiar explanations. While they could be true, it is quite hard to imagine being in a position to assert one if assertion requires knowledge and belief of what one is asserting.

Note also that order has no effect:

- (2) a. I don't know it's raining but it's raining
- b. I don't believe it's raining but it's raining.
- c. It might not be raining but it's raining.

Of course, this would change if we paused in between and learned new information. Nothing about these sentences seems to require any departure from standard semantic assumptions. In particular let me make this assumption about the meaning of "It might not be raining": namely that it is true iff "it's raining" is compatible with the speakers knowledge.

Yalcin [2007] showed that this simple view of things is not sustainable as an explanation of all such contradictions:

- (3) a. Suppose it's raining but I don't know it's raining.
- b. ? Suppose it's raining but it might not be raining.

Despite valiant efforts at pragmatic explanation of this contrast, most notably Dorr and Hawthorne [2014], it does not seem to me like we can explain the defiance of (3-b) without assuming something rather special about the semantics of 'might'. Yalcin gave his own semantics for 'might' but it was quickly recognized that his semantics is in most respects equivalent to the

earlier dynamic semantics of might proposed by Veltman [1996].¹ I won't rely here on the inadequacy of classical semantics to explain this as we'll see a novel problem.

Consider this sort of sentence:

- (4) a. ?Someone who is indoors might be outdoors.
- b. ?Someone who might be indoors is outdoors.
- c. ?Everyone who is indoors might not be .

Although Yalcin recently brought these to our attention (that is Nathan's and mine), it turns out there is a Dutch tradition of discussing this kind of sentence from Groenendijk et al. [1996] to Aloni [2001, chapter 3]. First of all, note that it's quite easy to get interpretations of 'might' in which it gets narrow scope under a quantifier:²

- (5) Every student in my class might be indoors, but at least one is outdoors.

So, there should be a reading of (4-a) which means roughly:

- (6) Someone is such that they are indoors but I do not know they are indoors.

Note that this is completely coherent, and (7) also seems coherent:

- (7) Someone who is indoors isn't known by me to be indoors.

Explaining what is happening here is our project. My conclusion will be, most importantly, that an explanation of what is happening requires a *dynamic* view of semantics.

There is a discussion to be had about the badness (or lack thereof) of sentences in (4). Given sufficient context such sentences are clearly comprehensible. What strikes me as requiring explanation is why they are not good without context.

¹Used oddly to explain the (alleged) order contrasts between (1-a) and (2-a).

²Perhaps contra von Stechow and Iatridou [2003].

Dynamics

There are two elements of the dynamic approach to semantics that are important due to Heim [1982] and Kamp [1981] on the one hand and Veltman on the other.

Files

Heim takes as the basic background information in a conversation not to be a set of possible worlds, but rather what she calls a file. A file consists of information about certain discourse referents.

The notion of discourse referent will not be *defined* directly, but rather we will see the role it plays in Heim. Nonetheless here are some starting thoughts.

(8) Scott or Alex met me at the train station. He was late.

He does not seem referential. Nonetheless by using ‘he’ we have introduced a discourse entity i.e. the person who came for dinner. See Karttunen [1976].

How do we model discourse referents? Let’s begin with the intuitive level take this discourse:

(9) A vegan sausage_{*x*} is on the plate. It_{*x*} is tasty. A man_{*y*} eats it_{*x*}.

Here we have two discourse referents, *x* and *y*, with respective information. . .

More formally, given a set *W* of worlds, and a set *O* of objects, and a set *V* of variables. We let *A* be a set of partial functions from *V* to *O*. A file is a set *F* of pairs of $\langle g, w \rangle$ ($g \in A, w \in W$) where for each $\langle g', w' \rangle$ and $\langle g'', w'' \rangle$ in *F*, *g* and *g'* share the same domain, called *F_D*. (In notion 1 it is just a set of assignment functions with the same domain).

Contexts thus are more articulated than Stalnakerian [1999] contexts (i.e. set of worlds): they contain not just worlds, but assignment functions as well. Files admit of an obvious notion of truth: a file *F* is true, iff there is a pair $\langle g, w \rangle$ in *F* such that *w* is actual.

Once we have files we can start to view semantics from the perspective of these. Sentences with indefinites ‘a woman walked in’ introduce new files

and add information about them. Sentences with pronouns and definites like ‘she is tall’ and ‘she smiled’ existing files.

Formal details:

$F[x] = \{\langle g, w \rangle : \exists \langle g', w \rangle \in F, g'[x]g\}$ (where $g'[x]g$ means that *g* and *g'* differ only in *x*).

if *x* is in domain of *x*, $F[Px] = \{\langle g, w \rangle \in F : g(x) \text{ satisfies } P \text{ in } w\}$ otherwise $F[Px] = \{\langle g, w \rangle \in F[x] : g(x) \text{ satisfies } P \text{ in } w\}$

$F[\phi \& \psi] = F[\phi][\psi]$

$F \ominus F' = \{\langle g, w \rangle \in F : \text{there is no } \langle g', w \rangle \in F' \text{ such that } g' \geq g\}$ (where $g' \geq g$ if *g'* agrees with *g* in the domain of *g*).

$F[\neg \phi] = F \ominus F[\phi]$

$F[\phi \rightarrow \psi] = F \ominus F[\phi][\neg \psi]$

Ugh. This kind of semantics gives reasonable treatments of these kinds of sentences:

(10) Everyone who owned a donkey beat it.

(11) A woman walked in. She sat down.

We assume that indefinites and pronouns are all expressions used to pick out variables. However, indefinites pick our variables not in the domain, whereas definite and pronoun pick out variables in the domain.

Epistemic Modality

There is a natural notion of epistemic modality to go with the idea of files. As a dialogue continues sentences update the discourse context. For something to possibly be true is simply for it to be compatible with the current discourse context. So for example, in a Stalnakerian context in which every world is a no-rain world, the sentence ‘it might be raining’ will not be acceptable.

Veltman formalized this idea as follows:

$F[\diamond \phi] = \{\langle g, w \rangle \in F : F[\phi] \neq \emptyset\}$

This means that the empty file is reached if there are no possibilities left by adding ϕ .

Applying dynamics to quantified epistemic contradictions

In dynamic semantics a sentence such as “Someone who is indoors might not be” has its deviance as a semantic matter. It essentially is an instruction to open a file x add the information that x is indoors to the file, and then check that the file is compatible with the information that x is not indoors.

What about the reverse order: “Someone who might not be indoors is indoors” this is more problematic (which perhaps explains a contrast some report). This is an instruction to open a file x , check that the information that x is not indoors is compatible with the file, and then add the information that x is indoors. This leaves you with a file in which x is indoors, so maybe it should be fine. We need a better notion of what makes a set of instructions acceptable. We’ll call this support: a set of update instructions S is supportable iff there is some file that is left unchanged by S . We can now see that these instructions while non-contradictory are unsupported. (Why might supportability track judgments of deviance? – much to say here, particularly involving knowledge/belief and norms of assertion).

Other dynamic approaches: markedly Groenendijk et al. [1996] does not get both orders correctly, since either is supportable. Aloni [2001] captures both orders (and was first to notice problem with other orders).

Other static approaches: as suggested, none in literature and possibility of one is doubtful (to me).

Wrinkles

All this seemed to give a good story of a narrow range of sentences, ones in which we use existential quantifiers. What about a wider range:

- (12) a. Most people who are indoors might not be.
b. Most people who might not be indoors are indoors.

Well known that we can’t treat ‘most’ as some logical combination of existential quantifier and negation, so we need a proper dynamic semantics of it.

If we also want to capture donkey anaphora our semantic entry must look something like this:

$$F[\text{Most}_x(\alpha, \beta)] = \{\langle g, w \rangle \in F : |\{a \in D : \exists g' \geq g \langle g'_{x \rightarrow a}, w \rangle \in F[\alpha]\}|/2 < |\{a \in D \exists g' \geq g \langle g'_{x \rightarrow a}, w \rangle \in F[\alpha][\beta]\}|^3$$

Ugh indeed, but this is practically the simplest semantics for ‘most’ that gives an adequate treatment of donkey anaphora [Chierchia, 1995]:

- (13) Most men who own a donkey beat it.

What’s interesting is on this entry (12-b) is supportable. Which seems to mean that even Aloni’s account of this phenomenon is inadequate.

What we seem to need here is to apply *locally* the constraint that successive updates be jointly supportable. In this case we immediately explain the badness of (12-b) and all other quantified epistemic quantifiers on the grounds that there is any quantifier on the dynamic story requires that its matrix predicate be evaluated in a context which already has its restrictor predicate. So those two together need to be supportable.⁴

Successive update rule:

if there is no F' such that $F'[\alpha] = F'$ and $F'[\beta] = F'$ then $F[\alpha][\beta] = \emptyset$, otherwise $F[\alpha][\beta] = (F[\alpha])[\beta]$

Pronouns and definites

So the story so far: there are some kinds of contradictions which reveal something about both how we keep track of variables in natural language and how epistemic modals work. In my view a pretty big argument for some aspects of the dynamic view of semantics (pending anyway a better explanation in a static framework—something I’ve tried and failed to find, or a rejection of the basic judgments).

Here are some further observations (Seth p.c., work-in-progress)

- (14) ?The man indoors might be outdoors.

³To make things a bit more complicated but more accurate we should really replace $F[\alpha]$ with $F_{x \rightarrow a}[\alpha]$ where $F_{x \rightarrow a}[\alpha] = \{\langle g_{x \rightarrow a}, w \rangle : \langle g, w \rangle \in F\}$

⁴Nathan and I give independent motivation for this principle elsewhere. In particular this principle is needed if you want to make sure that $\neg(\Diamond P \wedge \neg P)$ expresses a tautology/

This is actually very problematic as common views of definite descriptions and might give it these truth conditions:

- (15) There is a unique person who is indoors but I don't know that he is indoors.

Why should this sound odd? Our dynamic story applies directly here again since definites are just was of adding information to existing files (with complex stories about what we say about files that don't exist).

- (16) x is indoors, x might not be indoors.

However, note that similar problems arise with knowledge attributions:

- (17) I don't know that the man indoors is indoors.

Similar for some cases of intersentential anaphora:

- (18) A tall man is indoors. ? I don't know that he is indoors.

This does not seem equivalent to:

- (19) There is a tall man who is indoors who I don't know is indoors.

This is now a problem for the dynamic view, as on the dynamic view these sentences are equivalent. What is happening?

One idea: conversational presupposition of identifiability of definites and pronouns (in that sentences must not entail that referents are not identifiable).

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